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RYDBERG TERM TABLES*

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J. R. Rydberg[1] has calculated and used for series analysis a table of the values of $N/(m+a)^2$, which is exceedingly useful for practical series analysis. For the higher spectra of the atom (spark spectra) similar tables are desirable giving values of $Z^2N/(m+a)^2$ (Z = number of the spectrum; 1 for the arc spectrum, 2 for the first spark spectrum, 3 for the second spark spectrum, and so on. N = Rydberg's constant, which is best taken as $N_\infty = 109737.1$ cm^{-1}; m = order number of the term; a = Rydberg correction). Corresponding to the present needs of practical series analysis, tables are given to $Z=4$. (See Tables 1, 2, 3, and 4).

For the use of the tables, the following explanation is important: A succession of terms is represented by $Z^2N/(n-\Delta)^2$, where n is the real total quantum number and Δ is the quantum defect which is usually positive. In the tables, $m+a$ is used instead of $n-\Delta$. m then signifies a whole number, which is smaller than n by an integer which is usually one. If n is known, the relation between m and n is immediately given. For example, if $m=n-1$, then $\Delta = 1-a$.

For many purposes a term sequence may be sufficiently well represented by the expression of Ritz

$$(m,a) = \frac{Z^2N}{[m+a+\alpha(m,a)]^2}$$

Here a is the Rydberg, $\alpha(m, a)$ the Ritz correction. Now the term sequences of the higher terms, i.e., d-, f-, g-, sequences (due to non-

* [Translated by R. A. Sawyer. A limited number of reprints of this article are for sale by the Business Manager of the Journal for thirty cents each, in U. S. postage stamps.]

[1] J. R. Rydberg. Ostwalds Klassiker der exacten Wissenschaften. No. 196, p. 70.

TABLE 1. *Values of 109737.1/(m+a)² and the differences between m and m+1*

m	1	2	3	4	5	6	7	8	9	10	11	12	13
a													
+0.00	109737.1	27434.28	12193.01	6858.57	4389.48	3048.25	2239.53	1714.64	1354.78	1097.37	906.92	762.06	649.33
Δ		82302.82	15241.27	5334.44	2469.09	1341.23	808.72	524.89	359.86	257.41	190.45	144.86	112.73
+0.05	99534.78	26112.34	11796.52	6690.27	4302.99	2998.08	2207.88	1693.41	1339.85	1086.48	898.73	755.75	644.37
Δ		73422.44	14315.82	5106.25	2387.28	1304.91	790.20	514.47	353.56	253.37	187.75	142.98	111.38
+0.10	90691.82	24883.70	11419.05	6528.08	4219.04	2949.13	2176.89	1672.57	1325.17	1075.75	890.65	749.52	639.46
Δ		65808.12	13464.65	4890.97	2309.04	1269.91	772.24	504.32	347.40	249.42	185.10	141.13	110.06
+0.15	82977.04	23739.77	11059.42	6371.73	4137.51	2901.37	2146.55	1652.11	1310.72	1065.18	882.68	743.36	634.60
Δ		59237.27	12680.35	4687.69	2234.22	1236.14	754.82	494.44	341.39	245.54	182.50	139.32	108.76
+0.20	76206.33	22672.95	10716.51	6220.92	4058.33	2854.76	2116.84	1632.02	1296.52	1054.76	874.82	737.28	629.80
Δ		53533.38	11956.44	4495.59	2162.59	1203.57	737.92	484.82	335.50	241.76	179.94	137.54	107.48
+0.25	70231.75	21676.47	10389.31	6075.41	3981.39	2809.27	2087.75	1612.30	1282.54	1044.49	867.06	731.28	625.06
Δ		48555.28	11287.16	4313.90	2094.02	1172.12	721.52	475.45	329.76	238.05	177.43	135.78	106.22
+0.30	64933.18	20744.26	10076.87	5934.94	3906.63	2764.86	2059.24	1592.93	1268.78	1034.38	859.40	725.34	620.37
Δ		44188.92	10667.39	4141.93	2028.31	1141.77	705.62	466.31	324.15	234.40	174.98	134.06	104.97
+0.35	60212.39	19870.91	9778.31	5799.29	3833.95	2721.48	2031.32	1573.91	1255.25	1024.41	851.85	719.48	615.73
Δ		40341.48	10092.60	3979.02	1965.34	1112.47	690.16	457.41	318.66	230.84	172.56	132.37	103.75
+0.40	55988.32	19051.58	9492.83	5668.24	3763.28	2679.13	2003.96	1555.23	1241.93	1014.58	844.39	713.69	611.14
Δ		36936.74	9558.75	3824.59	1904.96	1084.15	675.17	448.73	313.30	227.35	170.19	130.70	102.55
+0.45	52193.63	18281.90	9219.67	5541.58	3694.54	2637.75	1977.16	1536.88	1228.82	1004.90	837.03	707.97	606.61
Δ		33911.73	9062.23	3678.09	1847.04	1056.79	660.59	440.28	308.06	223.92	167.87	129.06	101.36
+0.50	48772.04	17557.94	8958.13	5419.12	3627.67	2597.33	1950.88	1518.85	1215.92	995.35	829.77	702.32	602.12
Δ		31214.10	8599.81	3539.01	1791.45	1030.34	646.45	432.03	302.93	220.57	165.58	127.45	100.20

m	1	2	3	4	5	6	7	8	9	10	11	12	13
a													
+0.55	45676.21	16876.14	8707.56	5300.67	3562.60	2557.83	1925.13	1501.14	1203.22	985.94	822.60	696.73	597.69
Δ		28800.07	8168.58	3406.89	1738.07	1004.77	632.70	423.99	297.92	217.28	163.34	125.87	99.04
+0.60	42866.05	16233.30	8467.37	5186.06	3499.27	2519.22	1899.88	1483.74	1190.72	976.66	815.53	691.21	593.30
Δ		26632.75	7765.93	3281.31	1686.79	980.05	619.34	416.14	293.02	214.06	161.13	124.32	97.91
+0.65	40307.47	15626.50	8236.97	5075.13	3437.61	2481.48	1875.13	1466.63	1178.42	967.51	808.54	685.76	588.96
Δ		24680.97	7389.53	3161.84	1637.52	956.13	606.35	408.50	288.21	210.91	158.97	122.78	96.80
+0.70	37971.32	15053.10	8015.86	4967.73	3377.57	2444.58	1850.85	1449.82	1166.30	958.49	801.64	680.37	584.67
Δ		22918.22	7037.24	3048.13	1590.16	932.99	593.73	401.03	283.52	207.81	156.85	121.27	95.70
+0.75	35832.53	14510.69	7803.53	4863.69	3319.08	2408.50	1827.05	1433.30	1154.37	949.59	794.84	675.05	580.43
Δ		21321.84	6707.16	2939.84	1544.61	910.58	581.45	393.75	278.93	204.78	154.75	119.79	94.62
+0.80	33869.48	13997.08	7599.52	4762.89	3262.10	2373.21	1803.70	1417.06	1142.62	940.82	788.12	669.78	576.23
Δ		19872.40	6397.56	2836.63	1500.79	888.89	569.51	386.64	274.44	201.80	152.70	118.34	93.55
+0.85	32063.44	13510.26	7403.42	4665.20	3206.58	2338.69	1780.80	1401.09	1131.05	932.17	781.48	664.58	572.08
Δ		18553.18	6106.84	2738.22	1458.62	867.89	557.89	379.71	270.04	198.88	150.69	116.90	92.50
+0.90	30398.09	13048.41	7214.80	4570.47	3152.46	2304.92	1758.33	1385.39	1119.65	923.63	774.93	659.44	567.97
Δ		17349.68	5833.61	2644.33	1418.01	847.54	546.59	372.94	265.74	196.02	148.70	115.49	91.47
+0.95	28859.20	12609.84	7033.30	4478.61	3099.70	2271.87	1736.28	1369.96	1108.43	915.22	768.45	654.36	563.90
Δ		16249.36	5576.54	2554.69	1378.91	827.83	535.59	366.32	261.53	193.21	146.77	114.09	90.46
+1.00	27434.28	12193.01	6858.57	4389.48	3048.25	2239.53	1714.65	1354.78	1097.37	906.92	762.06	649.33	559.88
Δ		15241.27	5334.44	2469.09	1341.23	808.72	524.88	359.87	257.41	190.45	144.86	112.73	89.45
$m+a=$	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.50				
	121592.4	135477.9	151885.3	171464.2	195088.2	223953.4	259732.8	304825.9	438948.4				

TABLE 2,

Values of $4 \cdot 109737.1/(m+a)^3$ *and the differences between* m *and* $m+1$.

a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
.00	438948.40	109737.12	48772.04	27434.28	17557.92	12193.00	8958.12	6858.56	5419.12	4389.48	3627.68	3048.24	2597.32	2239.52	.00
	329211.28	60965.08	21337.76	9876.36	5364.92	3234.88	2099.56	1439.44	1029.64	761.80	579.44	450.92	357.80		
.05	398139.12	104449.36	47186.08	26761.08	17211.96	11992.32	8831.52	6773.64	5359.40	4345.92	3594.92	3023.00	2577.48	2223.70	.05
	293689.76	57263.28	20425.00	9549.12	5219.64	3160.80	2057.88	1414.24	1013.48	751.00	571.92	445.52	353.78		
.10	362767.28	99534.80	45676.20	26112.32	16876.16	11796.52	8707.56	6690.28	5300.68	4303.00	3562.60	2998.08	2557.84	2207.88	.10
	263232.48	53858.60	19563.88	9236.16	5079.64	3088.96	2017.28	1389.60	997.68	740.40	564.52	440.24	349.96		
.15	331908.16	94959.08	44237.68	25486.92	16550.04	11605.48	8586.20	6608.44	5242.88	4260.72	3530.72	2973.44	2538.40	2192.39	.15
	236949.08	50721.40	18750.76	8936.88	4944.56	3019.28	1977.76	1365.56	982.16	730.00	557.28	435.04	346.01		
.20	304825.32	90691.80	42866.04	24883.68	16233.32	11419.04	8467.36	6528.08	5186.08	4219.04	3499.28	2949.12	2519.20	2176.89	.20
	214133.52	47825.76	17982.36	8650.36	4814.28	2951.68	1939.28	1342.00	967.04	719.76	550.16	429.92	342.31		
.25	280927.00	86705.88	41557.24	24301.64	15925.56	11237.08	8351.00	6449.20	5130.16	4177.96	3468.24	2925.12	2500.24	2161.72	.25
	194221.12	45148.64	17255.60	8376.08	4688.48	2886.08	1901.80	1319.04	952.20	709.72	543.12	424.88	338.52		
.30	259732.72	82977.04	40307.48	23739.76	15626.52	11059.44	8236.96	6371.72	5075.12	4137.52	3437.60	2901.36	2481.48	2146.55	.30
	176755.68	42669.56	16567.72	8113.24	4567.08	2822.48	1865.24	1296.60	937.60	699.92	536.24	419.88	334.93		
.35	240849.56	79483.64	39113.24	23197.16	15335.80	10885.92	8125.28	6295.64	5021.00	4097.64	3407.40	2877.92	2462.92	2131.70	.35
	161365.92	40370.40	15916.08	7861.36	4449.88	2760.64	1829.64	1274.64	923.36	690.24	529.48	415.00	331.22		
.40	223953.28	76206.32	37971.32	22672.96	15053.12	10716.52	8015.84	6220.92	4967.72	4058.32	3377.56	2854.76	2444.56	2116.84	.40
	147746.96	38235.00	15298.36	7619.84	4336.60	2700.68	1794.92	1253.20	909.40	680.76	522.80	410.20	327.72		
.45	208774.52	73127.60	36878.68	22166.32	14778.16	10551.00	7908.64	6147.52	4915.28	4019.60	3348.12	2831.88	2426.44	2102.30	.45
	135646.92	36248.92	14712.36	7388.16	4227.16	2642.36	1761.12	1232.24	895.68	671.48	516.24	405.44	324.14		
a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	

a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
.50	195088.16	70231.76	35832.52	21676.48	14510.68	10389.32	7803.52	6075.40	4863.68	3981.40	3319.08	2809.28	2408.48	2087.75	.50
	124856.40	34399.24	14156.04	7165.80	4121.36	2585.80	1728.12	1211.72	882.28	662.32	509.80	400.80	320.73		
.55	182704.84	67504.56	34830.24	21202.68	14250.40	10231.32	7700.52	6004.56	4812.88	3943.76	3290.40	2786.92	2390.76	2073.50	.55
	115200.28	32674.32	13627.56	6952.28	4019.08	2530.80	1695.96	1191.68	869.12	653.36	503.48	396.16	317.26		
.60	171464.20	64933.20	33869.48	20744.24	13997.08	10076.88	7599.52	5934.96	4762.88	3906.64	3262.12	2764.84	2373.20	2059.24	.60
	106531.00	31063.72	13125.24	6747.16	3920.36	2477.36	1664.56	1172.08	856.24	644.52	497.28	391.64	313.96		
.65	161229.88	62506.00	32947.88	20300.52	13750.44	9925.92	7500.52	5866.52	4713.68	3870.04	3234.16	2743.04	2355.84	2045.28	.65
	98723.88	29558.12	12647.36	6550.08	3824.52	2425.40	1634.00	1152.84	843.64	635.88	491.12	387.20	310.56		
.70	151885.28	60212.40	32063.44	19870.92	13510.28	9778.32	7403.40	5799.28	4665.20	3833.96	3206.56	2721.48	2338.68	2031.32	.70
	91672.88	28148.96	12192.52	6360.64	3731.96	2374.92	1604.12	1134.08	831.24	627.40	485.08	382.80	307.36		
.75	143330.12	58042.76	31214.12	19454.76	13276.32	9634.00	7308.20	5733.20	4617.48	3798.36	3179.36	2700.20	2321.72	2017.64	.75
	85287.36	26828.64	11759.36	6178.44	3642.32	2325.80	1575.00	1115.72	819.12	619.00	479.16	378.48	304.08		
.80	135477.92	55988.32	30398.08	19051.56	13048.40	9492.84	7214.80	5668.24	4570.48	3763.28	3152.48	2679.12	2304.92	2003.96	.80
	79489.60	25590.24	11346.52	6003.16	3555.56	2278.04	1546.56	1097.76	807.20	610.80	473.36	374.20	300.96		
.85	128253.76	54041.04	29613.68	18660.80	12826.32	9354.76	7123.20	5604.36	4524.20	3728.68	3125.92	2658.32	2288.32	1990.51	.85
	74212.72	24427.36	10952.88	5834.48	3471.56	2231.56	1518.84	1080.16	795.52	602.76	467.60	370.00	297.81		
.90	121592.36	52193.64	28859.20	18281.88	12609.84	9219.68	7033.32	5541.56	4478.60	3694.52	3099.72	2637.76	2271.88	1977.16	.90
	69398.72	23334.44	10577.32	5672.04	3390.16	2186.36	1491.76	1062.96	784.08	594.80	461.96	365.88	294.70		
.95	115436.80	50439.36	28133.20	17914.44	12398.80	9087.48	6945.12	5479.84	4433.72	3660.88	3073.80	2617.44	2255.60	1964.02	.95
	64997.44	22306.16	10218.76	5515.64	3311.32	2142.36	1465.28	1046.12	772.84	587.08	456.36	361.84	291.58		
1.00	109737.12	48772.04	27434.28	17557.92	12193.00	8958.12	6858.56	5419.12	4389.48	3627.68	3048.24	2597.32	2239.52	1950.88	1.00
	60965.08	21337.76	9876.36	5364.92	3234.88	2099.56	1439.48	1029.64	761.80	579.44	450.92	357.80	288.64		
a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	

TABLE 3.

Values of $9 \cdot 1097.37.1/(m+a)^3$ and the differences between m and $m+1$.

a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	
.00	987633.90	246908.52	109737.09	61727.13	39505.32	27434.25	20155.77	15431.76	12193.02	9876.33	8162.28	6858.54	5843.97	.00
	740725.38	137171.43	48009.96	22221.81	12071.07	7278.48	4724.01	3238.74	2316.69	1714.05	1303.74	1014.57		
.05	895813.20	235011.06	106168.68	60212.43	38726.91	26982.72	19870.92	15240.69	12058.65	9778.32	8088.57	6801.75	5799.33	.05
	660801.96	128842.38	45956.25	21485.52	11744.19	7111.80	4630.23	3182.04	2280.33	1689.75	1286.82	1002.42		
.10	816226.38	223953.30	102771.45	58752.72	37971.36	26542.17	19592.01	15053.13	11926.53	9681.75	8015.85	6745.68	5755.14	.10
	592273.08	121181.85	44018.73	20781.36	11429.19	6950.16	4538.88	3126.60	2244.78	1665.90	1270.17	990.54		
.15	746793.36	213657.93	99534.78	57345.57	37237.59	26112.33	19318.95	14868.99	11796.48	9586.62	7944.12	6690.24	5711.40	.15
	533135.43	114123.15	42189.21	20107.98	11125.26	6793.38	4449.96	3072.51	2209.86	1642.50	1253.88	978.84		
.20	685856.97	204056.55	96448.59	55988.28	36524.97	25692.84	19051.56	14688.18	11668.68	9492.84	7873.38	6635.52	5668.20	.20
	481800.46	107607.96	40460.31	19463.31	10832.13	6641.28	4363.38	3019.50	2175.84	1619.46	1237.86	967.32		
.25	632085.76	195088.23	93503.79	54678.69	35832.51	25283.43	18789.75	14510.70	11542.86	9400.41	7803.54	6581.52	5625.54	.25
	436997.52	101584.44	38825.10	18846.18	10549.08	6493.68	4279.05	2967.84	2142.45	1596.87	1222.02	955.98		
.30	584398.62	186698.34	90691.83	53414.46	35159.67	24883.74	18533.16	14336.37	11419.02	9309.42	7734.60	6528.06	5583.33	.30
	397700.28	96006.51	37277.37	18254.79	10275.93	6350.58	4196.79	2917.35	2109.60	1574.82	1206.54	944.73		
.35	541911.51	178838.19	88004.79	52193.61	34505.55	24493.32	18281.88	14165.19	11297.25	9219.69	7666.65	6475.32	5541.57	.35
	363073.32	90833.40	35811.18	17688.06	10012.23	6211.44	4116.69	2867.94	2077.56	1553.04	1191.33	933.75		
.40	503894.88	171464.22	85435.47	51014.16	33869.43	24112.17	18035.64	13997.07	11177.37	9131.22	7599.51	6423.21	5500.26	.40
	332430.66	86028.75	34421.31	17144.73	9757.26	6076.53	4038.57	2819.70	2046.15	1531.71	1176.30	922.95		
.45	469742.67	164537.10	82977.03	49874.22	33250.86	23739.75	17794.44	13831.92	11059.38	9044.10	7533.27	6371.73	5459.49	.45
	305205.57	81560.07	33102.81	16623.36	9511.11	5945.31	3962.52	2772.54	2015.28	1510.83	1161.54	912.24		
a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	

a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	
.50	438948.36	158021.46	80623.17	48772.08	32649.03	23375.97	17557.92	13669.65	10943.28	8958.15	7467.93	6320.88	5419.08	.50
	280926.90	77398.29	31851.09	16123.05	9273.06	5818.05	3888.27	2726.37	1985.13	1490.22	1147.05	901.80		
.55	411085.89	151885.26	78368.04	47706.03	32063.40	23020.47	17326.17	13510.26	10828.98	8873.46	7403.40	6270.57	5379.21	.55
	259200.63	73517.22	30662.01	15642.63	9042.93	5694.30	3815.91	2681.28	1955.52	1470.06	1132.83	891.36		
.60	385794.45	146099.70	76206.33	46674.54	31493.43	22672.98	17098.92	13353.66	10716.48	8789.94	7339.77	6220.89	5339.70	.60
	239694.75	69893.37	29531.79	15181.11	8820.45	5574.06	3745.26	2637.18	1926.54	1450.17	1118.88	881.19		
.65	362767.23	140638.50	74132.73	45676.17	30938.49	22333.32	16876.17	13199.67	10605.78	8707.59	7276.86	6171.84	5300.64	.65
	222128.73	66505.77	28456.56	14737.68	8605.17	5457.15	3676.50	2593.89	1898.19	1430.73	1105.02	871.20		
.70	341741.88	135477.90	72142.74	44709.57	30398.13	22001.22	16657.65	13048.38	10496.70	8626.41	7214.76	6123.33	5262.03	.70
	206263.98	63335.16	27433.17	14311.44	8396.91	5343.57	3609.27	2551.68	1870.29	1411.65	1091.43	861.30		
.75	322492.77	130596.21	70231.77	43773.21	29871.72	21676.50	16443.45	12899.70	10389.33	8546.31	7153.56	6075.45	5223.87	.75
	191896.56	60364.44	26458.56	13901.49	8195.22	5233.05	3543.75	2510.37	1843.02	1392.75	1078.11	851.58		
.80	304825.32	125973.72	68395.68	42866.01	29358.90	21358.89	16233.30	12753.54	10283.58	8467.38	7093.08	6028.02	5186.07	.80
	178851.60	57578.04	25529.67	13507.11	8000.01	5125.59	3479.76	2469.96	1816.20	1374.30	1065.06	841.95		
.85	288570.96	121592.34	66630.78	41986.80	28859.22	21048.21	16027.20	12609.81	10179.45	8389.53	7033.32	5981.22	5148.72	.85
	166978.62	54961.56	24643.98	13127.58	7811.01	5021.01	3417.39	2430.36	1789.92	1356.21	1052.10	832.50		
.90	273582.81	117435.69	64933.20	41134.23	28372.14	20744.28	15824.97	12468.51	10076.85	8312.67	6974.37	5934.96	5111.73	.90
	156147.12	52502.49	23798.97	12762.09	7627.86	4919.31	3356.46	2391.66	1764.18	1338.30	1039.41	823.23		
.95	259732.80	113488.56	63299.70	40307.49	27897.30	20446.83	15626.52	12329.64	9975.87	8236.98	6916.05	5889.24	5075.10	.95
	146244.24	50188.86	22992.21	12410.19	7450.47	4820.31	3296.88	2353.77	1738.89	1320.93	1026.81	814.14		
1.00	246908.52	109737.09	61727.13	39505.32	27434.25	20155.77	15431.76	12193.02	9876.33	8162.28	6858.54	5843.97	5038.92	1.00
	137171.43	48009.96	22221.81	12071.07	7278.48	4724.01	3238.74	2316.69	1714.05	1303.74	1014.57	805.05		
a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	

penetrating electron orbits) are distinctly different in their progression from the term sequences of s-and p-terms (due to penetrating orbits). In both cases the quantity $\alpha(m, a)$ decreases with increasing m, so that the term sequence approaches $Z^2N/(m+a)^2$ asymptotically. But for lower values of m there is a difference which is shown by the sign of the quantity $\alpha(m, a)$. α is positive for non-penetrating and negative for penetrating orbits. In any term sequence the terms of higher values of m in general lie between two horizontal rows of the table. Then the terms of lower m values lie below, in the case of non-penetrating orbits; in the case of penetrating orbits, they lie above these rows, as Fig. 1 shows. For any given term (m, a) the appropriate value of a can be found from the table since it is possible to interpolate linearly between two horizontal rows.

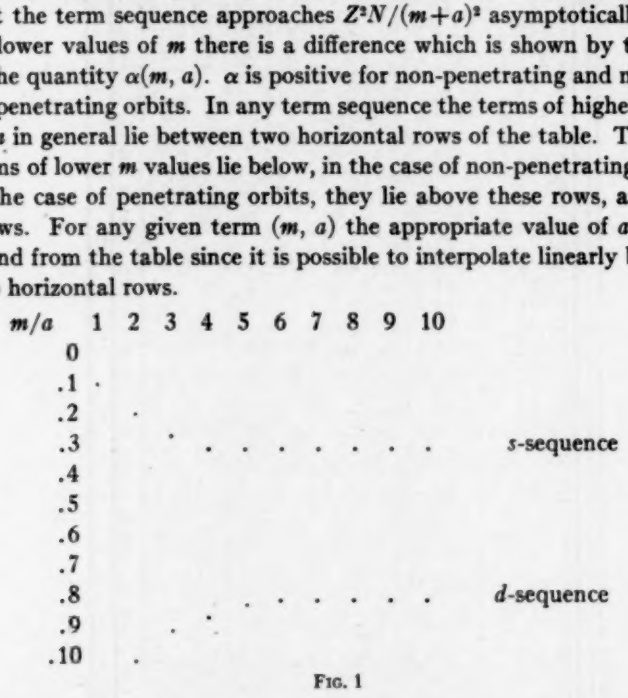

FIG. 1

With the aid of the tables it is possible to verify the members of a series and also to determine the limit A of the series. [The lines of a series are given by $\nu = A - (m, a)$.] The wave number differences between successive lines of a series are differences of the value of (m, a) for successive values of m. These differences for large values of m must give a value of a approaching a constant. In the following, two different series will be discussed on the basis of the table and their limits calculated.

Example I. See Table 5. Helium "doublets," Stronger component $2^3P_{1,2} - mS$. $\nu_{observed} + (m, a)_{table}$ must for large values of m give the value $2P$ of the limit. A constant value must thus be computed for higher values of m. In the example $2P$ is not constant but decreases with increasing m. This is an indication that the value $a = 0.70$ from the table is not the

correct one. For an approximate calculation of the limit the agreement is, however, sufficient. About 29224.0 would be a more correct limit. The following procedure will give a more exact calculation of the limit: Assume the value 29224.0; from this value and the observed values of ν, compute values of (m, a) and from them the quantities $m^* = \sqrt{N}/(m,a)$. According to Ritz, these must be $m^* = m + a + \alpha(m^*, a)$ and since $\alpha(m^*, a)$ does not enter into consideration for larger values of m, these quantities must approach $m + a$ asymptotically with increasing m. With 29224.0 this is not sufficiently near the case. Also the values of m^* obtained from this value are not well represented by the Ritz expression. It is found by trial that both these conditions are better satisfied with the value of 29223.87 for the limit. With this value of $2P$ the following values of (m, a) and of m^* are obtained (calculated with $N_{H_e} = 109722.14$). See Table 6.

The last three values show a further deviation. This would vanish if the value $2P$ and the accompanying values (m, a) were diminished still further by 0.05. The chosen value of $2P$ gives a good representation of m^* for $m = 2$ to $m = 6$ by the Ritz formula. If in the expression of m^* Hick's expression α/m were used in place of Ritz's $\alpha(m, a)$, the last correction would be different. Hick's expression has, however, little physical significance. According to Bohr's model, it is sufficient if an asymptotic approximation to $m + a$ is attained, which in the preceding example would lead to the value of 29223.87.

In the term sequence (m, a), there is still the term $(1, a)$ which results from the line $\lambda_{air} = 10830.32$ AU., $\nu = 2S - 2P = 9230.81$. From this we get $(1, s) = 38454.68$. $1^* = 1.68917$.

It is evident that the values of m^* become smaller with increasing m, or that the larger values of (m, a) move into higher lying rows of the table. The term sequence corresponds to penetrating orbits. According to Bohr the total quantum number n is equal to $m + 1$ and in $n - \Delta$ the quantum defect decreases with increasing n and approaches asymptotically the value $\Delta = 0.297$.

Example II. See Table 7, Mg I, $2^3P_2 - m^3D$

The values approach the horizontal row $a = 0.825$ of the table asymptotically, while for lower values of m they move into lower horizontal rows (non-penetrating orbits). The Bohr total quantum number is equal to $m + 1$ and the quantum defect $\Delta = 1 - a = 0.175$.

The tables for values of Z larger than 1 can be used in the same way. They save division of the wave numbers by Z^2, which would be necessary for use of the table of $N/(m + a)^2$. Since in searching for series a

TABLE 4.

Values of $16 \cdot 109737.1/(m+a)^2$ and the differences between m and $m+1$.

a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	
.00	1755793.6	438948.5	195088.2	109737.1	70231.7	48772.0	35832.5	27434.2	21676.5	17557.9	14510.7	12193.0	10389.3	.00
	1316845.1	243860.3	85351.1	39505.4	21459.7	12939.5	8398.3	5757.7	4118.6	3047.2	2317.7	1803.7		
.05	1592556.5	417797.4	188744.3	107044.3	68847.8	47969.3	35326.1	27094.6	21437.6	17383.7	14379.7	12092.0	10309.9	.05
	1174759.1	229053.1	81700.0	38196.5	20878.5	12643.2	8231.5	5657.0	4053.9	3004.0	2287.7	1782.1		
.10	1451069.1	398139.2	182704.8	104449.3	67504.6	47186.1	34830.2	26761.1	21202.7	17212.0	14250.4	11992.3	10231.4	.10
	1052929.9	215434.4	78255.5	36944.7	20318.5	12355.9	8069.1	5558.4	3990.7	2961.6	2258.1	1760.9		
.15	1327632.6	379836.3	176950.7	101947.7	66200.2	46421.9	34344.8	26433.8	20971.5	17042.9	14122.9	11893.8	10153.6	.15
	947796.3	202885.6	75003.0	35747.5	19778.3	12077.1	7911.0	5462.3	3928.6	2920.0	2229.1	1740.2		
.20	1219301.3	362767.2	171464.2	99534.7	64933.3	45676.2	33869.4	26112.3	20744.3	16876.2	13997.1	11796.5	10076.8	.20
	856534.1	191303.0	71929.5	34601.4	19257.1	11806.8	7757.1	5368.0	3868.1	2879.1	2200.6	1719.7		
.25	1123708.0	346823.5	166229.0	97206.6	63702.2	44948.3	33404.0	25796.8	20520.6	16711.8	13873.0	11700.5	10001.0	.25
	776884.5	180594.5	69022.4	33504.4	18753.9	11544.3	7607.2	5276.2	3808.8	2838.8	2172.5	1699.5		
.30	1038930.9	331908.2	161229.9	94959.0	62506.1	44237.8	32947.8	25486.9	20300.5	16550.1	13750.4	11605.4	9925.9	.30
	707022.7	170678.3	66270.9	32452.9	18268.3	11290.0	7460.9	5186.4	3750.4	2799.7	2145.0	1679.5		
.35	963398.2	317934.6	156453.0	92788.6	61343.2	43543.7	32501.1	25182.6	20084.0	16390.6	13629.6	11511.7	9851.7	.35
	645463.6	161481.6	63664.4	31445.4	17799.5	11042.6	7318.5	5098.6	3693.4	2761.0	2117.9	1660.0		
.40	895813.1	304825.3	151885.3	90691.8	60212.5	42866.1	32063.4	24883.7	19870.9	16233.3	13510.2	11419.0	9778.2	.40
	590987.8	152940.0	61193.5	30479.3	17346.4	10802.7	7179.7	5012.8	3637.6	2723.1	2091.2	1640.8		
.45	835098.1	292510.4	147514.7	88665.3	59112.6	42204.0	31634.6	24590.1	19661.1	16078.4	13392.5	11327.5	9705.8	.45
	542587.7	144995.7	58849.4	29552.7	16908.6	10569.4	7044.5	4929.0	3582.7	2685.9	2065.0	1621.7		
a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	

a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	
.50	780352.6	280927.0	143330.1	86705.9	58042.7	41557.3	31214.1	24301.6	19454.7	15925.6	13276.3	11237.1	9633.9	.50
	499425.6	137596.9	56624.2	28663.2	16485.4	10343.2	6912.5	4846.9	3529.1	2649.3	2039.2	1603.2		
.55	730819.4	270018.2	139321.0	84810.7	57001.6	40925.3	30802.1	24018.2	19251.5	15775.0	13161.6	11147.7	9563.0	.55
	460801.2	130697.2	54510.3	27809.1	16076.3	10123.2	6783.9	4766.7	3476.5	2613.4	2013.9	1584.7		
.60	685856.8	259732.8	135477.9	82977.0	55988.3	40307.5	30398.1	23739.8	19051.5	15626.6	13048.5	11059.4	9492.8	.60
•	426124.0	124254.9	52500.9	26988.7	15680.8	9909.4	6658.3	4688.3	3424.9	2578.1	1989.1	1566.6		
.65	644919.5	250024.0	131791.5	81202.1	55001.8	39703.7	30002.1	23466.1	18854.7	15480.2	12936.6	10972.2	9423.4	.65
	394895.5	118232.5	50589.4	26200.3	15298.1	9701.6	6536.0	4611.4	3374.5	2543.6	1964.4	1548.8		
.70	607541.1	240849.6	128253.8	79483.7	54041.1	39113.3	29613.6	23197.1	18660.8	15335.8	12826.2	10885.9	9354.7	.70
	366691.5	112595.8	48770.1	25442.6	14927.8	9499.7	6416.5	4536.3	3325.0	2509.6	1940.3	1531.2		
.75	573320.5	232171.0	124856.5	77819.0	53105.3	38536.0	29232.8	22932.8	18469.9	15193.4	12717.4	10800.8	9286.9	.75
	341149.5	107314.5	47037.5	24713.7	14569.3	9303.2	6300.0	4462.9	3276.5	2476.0	1916.6	1513.9		
.80	541911.7	223953.3	121592.3	76206.2	52193.6	37971.4	28859.2	22673.0	18281.9	15053.1	12609.9	10716.5	9219.7	.80
	317958.4	102361.0	45386.1	24012.6	14222.2	9112.2	6186.2	4391.1	3228.8	2443.2	1893.4	1496.8		
.85	513015.0	216164.2	118454.7	74643.2	51305.3	37419.0	28492.8	22417.4	18096.8	14914.7	12503.7	10633.3	9153.3	.85
	296850.8	97709.5	43811.5	23337.9	13886.3	8926.2	6075.4	4320.6	3182.1	2411.0	1870.4	1480.0		
.90	486369.4	208774.6	115436.8	73127.5	50439.4	36878.7	28133.3	22166.2	17914.4	14778.1	12398.9	10551.0	9087.5	.90
	277594.8	93337.8	42309.3	22688.1	13560.7	8745.4	5967.1	4251.7	3136.3	2379.2	1847.9	1463.5		
.95	461747.2	201757.4	112532.8	71657.8	49595.2	36349.9	27780.5	21919.4	17734.9	14643.5	12295.2	10469.8	9022.4	.95
	259989.8	89224.6	40875.0	22062.6	13245.3	8569.4	5861.1	4184.5	3091.4	2348.3	1825.4	1447.4		
1.00	438948.5	195088.2	109737.1	70231.7	48772.0	35832.5	27434.2	21676.5	17557.9	14510.7	12193.0	10389.3	8958.1	1.00
	243860.3	85351.1	39505.4	21459.7	12939.5	8398.3	5757.7	4118.6	3047.2	2317.7	1803.7	1431.2		
a/m	1	2	3	4	5	6	7	8	9	10	11	12	13	

TABLE 5.

Observed	λ_{air}	7065.200	4713.143	4120.817	3867.477	3732.861	3651.981	3599.304	3562.950	3536.820	3517.327	3502.381
	ν	14149.98	21211.34	24260.20	25849.33	26781.50	27374.61	27775.24	28058.63	28265.92	28422.56	28543.85
	Δ	7061.36	3048.86	1589.13	932.17	593.11	400.63	283.39	207.29	156.64	121.29	
Table $a=0.70$	Δ	7037.24	3048.13	1590.17	932.98	593.73	401.03	283.52	207.81	156.85	121.27	
	(m,a)	—15053.10	8015.86	4967.73	3377.56	2444.58	1850.85	1449.82	1166.30	958.49	801.64	680.37
	m	2	3	4	5	6	7	8	9	10	11	12
$2P$ calc		29203.08	29227.20	29227.93	29226.89	29226.08	29225.46	29225.06	29224.93	29224.41	29224.20	29224.22

TABLE 6.

m (table)	2	3	4	5	6	7	8	9	10	11	12
(m,a)	15073.89	8012.53	4963.67	3374.54	2442.37	1849.26	1448.63	1165.24	957.95	801.31	680.02
$*m$	2.697954	3.700517	4.701601	5.70217	6.70257	7.70279	8.70299	0.70375	10.7023	11.7017	12.7024

TABLE 7

Observed	λ_{air}	3838.29	3096.91	2851.65	2736.53	2672.43	2632.88	2606.64	2588.28	2574.93	2564.91	2557.20	2551.13
	ν	26045.9	32280.9	35057.1	36531.8	37408.1	37970.0	38352.1	38624.2	38824.4	38976.0	39093.5	39186.5
	Δ	6235.0	2776.2	1474.7	876.3	561.9	382.1	272.1	200.2	151.6	117.5	93.0	
Table $a=0.825$	Δ	6252.20	2787.42	1470.71	878.39	563.70	383.17	272.24	200.34	151.70	117.62	93.03	
	(m,a)	13753.67	7501.47	4714.05	3234.34	2355.95	1792.25	1409.08	1136.84	936.50	784.80	667.18	574.15
		2	3	4	5	6	7	8	9	10	11	12	13
2^3P_1		39799.57	39782.37	39771.15	39766.14	39764.05	39762.25	39761.18	39761.04	39760.9	39760.8	39760.7	39760.65
39760.5 may be taken as an asymptotic value of 2^3P_1 for large values of m. From this value there follows for (m, D):													
$m=$	2	3	4	5	6	7	8	9	10	11	12	13	
(m,D)	13714.6	7479.6	4703.4	3228.7	2352.4	1790.5	1408.4	1136.3	936.1	784.5	667.0	574.0	

great many such divisions are necessary, the simplification is great. In fact, in the investigation of the spectrum of Al III, I divided the wave numbers of all the lines by 9; I first calculated a table of the values of $4N/(m+a)^2$ as a most necessary tool for the investigation of some of the first spark spectra.

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The tables for $Z=1$ and $Z=2$ were computed by Professor Paschen. With his consent we have computed those for $Z=3$ and $Z=4$ by multiplication of the table for $Z=1$. All the tables were checked against one another in two ways. First, all the differences between terms obtained by multiplication were checked by actual subtraction. Second, since for example, $N/(1.15)^2=4N/(2.30)^2=9N/(3.45)^2=16N/(4.60)^2$, every other term value in the $Z=2$ table, every third term in the $Z=3$ table, and every fourth term in the $Z=4$ table, may be independently checked. In this way any error of as much as .01 in the $Z=1$ table should be detected. Obviously since the other tables are obtained by multiplication of this table which may be in error by .005, the $Z=2$ table may be in error by .02, the $Z=3$ table by .05, and the $Z=4$ table by .08. For this reason the $Z=4$ table has been given to one less place than the other tables.

The table for $Z=1$ is that of Paschen-Götze: "Seriengesetze der Linienspektren." It is reprinted here both for the sake of completeness and for uniformity since several errors in it are here corrected.

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THE DEPENDENCE OF THE RESOLVING POWER OF A PHOTOGRAPHIC MATERIAL UPON THE CONTRAST IN THE OBJECT*

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ABSTRACT

The resolving power of a photographic material may be defined qualitatively as the ability to show fine detail in the picture. It is defined numerically as the number of lines and spaces per millimeter which it resolves. This definition, however, is rather inadequate since resolving power depends on many factors, such as the ratio of the width of the line to the width of the space, the color temperature of the light image, or the wavelength where monochromatic radiation is in question, and the contrast in the object.

The present paper gives some results of an experimental investigation of the dependence of the resolving power upon the contrast in the test object, where contrast is defined as the ratio of the photographic intensities of two adjacent small images to be resolved.

The method of investigation was to photograph in a reducing camera a series of parallel line test objects differing only in contrast, and by microscopic exmaination of the developed photographic images to determine the maximum resolving power for the respective objects.

The results show that the resolution changes very rapidly with contrast at low contrast values. Thus, with no resolution at unit contrast, the resolving power reaches approximately 65% of its maximum value for a test object density of 0.5, that is, a transmission through the opaque spaces of 31.5% or a contrast of 3.17; and 87% of its maximum value when the test object density is 1.0, transmission 10% and contrast is 10. The maximum value of resolving power is reached when the test object has a contrast of approximately 100 to 200.

The resolving power of a photographic material may be stated qualitatively as the ability to show fine detail in the picture. It is defined quantitatively as the number of lines and spaces per millimeter which it resolves. This definition, however, is inadequate, since the resolving power depends on several factors, such as the ratio of the width of the line to the width of the opaque space;[1] the spectral composition of the image, or the wavelength where monochromatic radiation is in question; and the contrast in the object.

The present paper gives some results of an experimental investigation of the dependence of the resolving power on the contrast in the object; where the contrast is defined as the ratio of the photographic intensities of two adjacent small images to be resolved.[2] As a matter of convenience, we have chosen the simplest case, namely, an object of definite form, non-selective and uniformly illuminated by tungsten radiation

* Communication No. 334 from the Kodak Research Laboratories.

[1] Otto Sandvik. On the Measurement of the Resolving Power of Photographic Materials, J.O.S.A. & R.S.I., *14:* p. 169; 1927.

[2] L. A. Jones. The Contrast of Photographic Printing Papers. J. Frank. Inst., *202*, p. 177, 1926; *203*, p. 111, 1927.

screened to daylight quality. These conditions simulate approximately those encountered in the process of photographic sound recording on motion picture film where the variation in contrast is brought about by changing the intensity of the light or the total exposure without altering the quality of the radiation. The separation of a close pair of spectrum lines is the same type of problem. In connection with color photography the problem is more complex because two adjacent areas may differ not only in photographic intensity but in color also. This phase of the problem is now being investigated by a study of the dependence of resolving power on the wave lengths in addition to changes in resolving power due to contrast.

The method employed in the present investigation was to photograph in a reducing camera a series of parallel line test objects shown in Fig. 1. This consists of twenty-five groups of parallel lines; each group

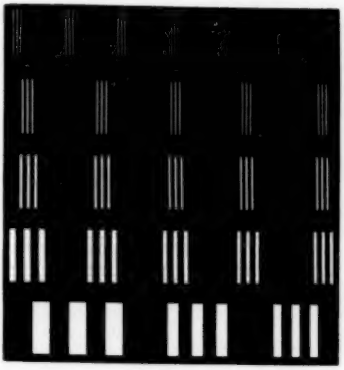


FIG. 1. *Parallel line test object.*

contains two opaque spaces and three transparent lines; the width of the lines and the width of the spaces are equal in each group. The distance between the center of two adjacent lines or the center of two adjacent spaces will be designated by d. The distance d varies from 0.13 to 2 mm.

The reduction in the camera is 20 diameters, hence the resolving power R, in terms of lines per millimeter is given by the equation,

$$R=\frac{1}{d}\cdot\frac{1}{0.05}.$$

Thus the range of resolving power values covered by the groups of lines on this object when photographed in this camera at a reduction of

20 diameters will be from 1/0.13 ·1/0.05 to 1/2 ·1/0.05, that is from 154 to 10.

Ten test objects, such as that shown in Fig. 1, equal in size but varying in contrast, were used. As a matter of convenience we shall specify contrast of the test objects in terms of D, the density of the partially opaque spaces and background. We shall define contrast, C, as the ratio of the intensity of the radiation being transmitted by unit area of the clear sections to that transmitted through unit area of the opaque sections, that is, $C=T_c/T_0$.

Where T_c and T_0 be the transmissions of the clear and the opaque sections respectively, and the transmission, T, is defined as the ratio between the intensity of the transmitted to the incident radiation, that is T_1/T_2. But the density, D, is

$$D=\log T_2/T_1=\log 1/T\cdot \tag{1}$$

Hence D_c and D_0 the densities of the clear and the opaque sections respectively are

$$D_c=\log 1/T_c \tag{a}$$

and

$$D_0=\log 1/T_0$$

But by definition the contrast, C, is

$$C=T_c/T_0 \tag{2}$$

hence

$$(\log 1/T_0)/(\log 1/T_c)=\log C=D_0-D_c. \tag{3}$$

Now, since the clear sections have a negligible amount of absorption due to photographic grain deposits, we may call T_c unity, therefore, $D_c=0$ and

$$\log C=D_0. \tag{4}$$

In the following pages we shall drop the subscript of D_0 and let $(D_0\equiv D)$. That is whenever we refer to the test object density, D, the density of the opaque sections will be understood.

The density of the ten test objects used ranged from 0.25 to 4.0, corresponding to a contrast ratio ranging from 1.75 to 10,000.

Four series of exposures were made through each test object and developed to four different gammas. The consecutive exposure times were increased by powers of two from ¼ second to 1024 seconds. Then by microscopic examination of the series of images resulting from these developed exposures the maximum resolving power was determined for the various values of D.

It may be well at this point to consider briefly the change of resolving power with exposure. It is agreed quite generally that resolution increases with decreasing exposures. The percentage change, however, is much larger than the values usually found in the literature. C. E. K. Mees[3] has pointed out the important rôle which the turbidity of the emulsion plays in connection with resolving power. Undoubtedly the decrease in the resolution on exposure is due to a spreading of the image produced by light scattered in the emulsion. This spreading will continue so long as the exposure does. This is indicated by the fact that for a series of exposures whose developed densities lie on the straight line of the characteristic curve, the resolving power is approximately inversely proportional to the density and hence to logarithm of the exposure. When a certain minimum value of exposure is reached,

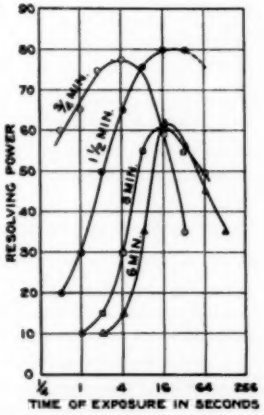


FIG. 2. *Curves showing the change of the resolving power with the time of exposure for an Eastman Process Plate. The time of development in minutes is indicated on each curve.*

however, the resolving power diminishes very rapidly. There are two very obvious reasons why resolution should fall off at low values of exposure; first, because only a few scattered grains are rendered developable, thus making it difficult to distinguish these from grains due to fog; and secondly, because the image formed by low exposure falls on underexposed portion of the characteristic curve where the value of gamma, and therefore the subjective contrast, is very small. The results

[3] C. E. K. Mees. On the Resolving Power of the Photographic Plates. Proc. Roy. Soc., *83A*: p. 10; 1909. The Physics of the Photographic Process. J. Frank. Inst., *179*: p. 141, 1915.

obtainable on this particular phase of the problem are erratic, making it difficult to find a functional relationship between resolution and exposure.

Fig. 2 shows the change of resolving power of an Eastman Process plate for different exposures through a test object whose density was 2.5. The four curves are four different times of development as indicated in the figure. This is a fairly representative case of the relationship between the two quantities, that is, with decreasing exposures the resolving power grows more or less linearly to a maximum value whence it falls off very rapidly.

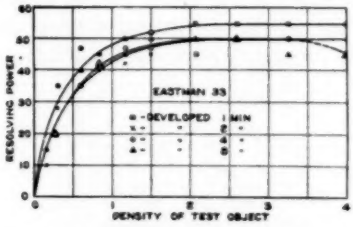


FIG. 3. *Curves showing the resolving power for developments to different values of gamma. The different values of gamma are given at the head of Table 1.*

According to existing theories for a given set of exposure conditions the resolving power is a linear function of gamma. *A priori* this is

TABLE 1. *The dependence of resolving power on the density, D, of the opaque sections of the test object where D=log C and C is the contrast of the test object. The mean values of resolving power for the four different values of gamma, are listed in the last column.*

Emulsion	Density of test object	Resolving power for different values for gamma				Mean value of resolving power
		$\gamma=0.57$	$\gamma=0.95$	$\gamma=1.48$	$\gamma=1.89$	
Eastman 33	0.16	15	10	15	15	14
	0.30	35	28	25	25	28
	0.60	47	35	35	40	39
	0.83	45	40	42	42	42
	1.17	50	42	47	45	46
	1.50	52	45	47	50	49
	2.07	55	50	45	50	50
	2.60	55	50	50	50	51
	3.05	55	50	50	45	50
	4.00	55	50	45	45	49

certainly what one would expect. The author has failed to find any systematic variation, however, although it is more probable that resolving power decreases with development as shown in Table 1 and Fig. 3. These give the resolving power of an Eastman 33 emulsion for the ten different test objects when developed in a standard M.Q. Process developer at 68°F for 1, 2, 4, and 8 minutes with the resulting gammas of 0.57, 0.95, 1.48, and 1.89 respectively. In this case there is a definite

TABLE 2. *A comparison of the averages of the observed values of R in the last column of Table 1 and the corresponding values of R computed from formula (5).*

Emulsion	C	D	$R_{obs.}$	$R_{calc.}$	
Eastman 33	51	0	—	0	—
		0.09	10	9.7	1.04
		0.20	20	19	1.07
		0.36	30	29	1.07
		0.48	35	34.3	1.05
		0.67	40	40.3	0.99
		1.03	45	46.3	0.90
		1.75	50	50.1	0.97
		2.50	51	50.9	—
			—	51	—
				Mean	1.01

TABLE 3. *The dependence of the resolving power on the density, D, of the opaque sections of the test object. Where D=log C and C is the contrast of the test object. The tables are for Eastman Process, Eastman 40, Eastman Speedway, and W & W Panchromatic plates respectively.*

Emulsion	Density of test object	Resolving power for different values of gamma				Mean value of resolving power
		$\gamma=0.93$	$\gamma=1.82$	$\gamma=2.50$	$\gamma=3.10$	
Eastman Process	0.16	35	40	15	35	29
	0.30	45	40	40	40	41
	0.60	70	50	45	45	52
	0.83	70	56	60	55	60
	1.17	75	75	60	57	67
	1.50	65	65	70	59	70
	2.07	75	77	72	60	71
	2.60	75	75	75	60	71
	3.05	80	80	60	60	70
	4.00	70	75	75	60	70

decrease in the resolution for longer times of development. The same is true for the other four emulsions as shown by Tables 3, 4, 5, and 6, and is particularly pronounced for the Wratten Panchromatic, Table 6.

No doubt if all conditions could be fixed except the time of development then resolution would grow with gamma. This not being the

TABLE 4. *The dependence of the resolving power on the density, D, of the opaque sections of the test object. Where D=log C and C is the contrast of the test object. The tables are for Eastman Process, Eastman 40, Eastman Speedway, and W & W Panchromatic plates respectively.*

Emulsion	Density of test object	Resolving power for different values of gamma				Mean value of resolving power
		$\gamma=0.55$	$\gamma=0.96$	$\gamma=1.21$	$\gamma=1.58$	
Eastman 40	0.16	10	10	10	10	10
	0.30	20	20	20	20	20
	0.60	30	35	30	30	31
	0.83	35	40	35	35	36
	1.17	40	35	40	39	38
	1.50	40	40	40	35	39
	2.07	40	35	40	40	39
	2.60	40	40	40	40	40
	3.05	40	35	40	40	39
	4.00	30	40	35	40	36

TABLE 5. *The dependence of the resolving power on the density, D, of the opaque sections of the test object. Where D=log C and C is the contrast of the test object. The tables are for Eastman Process, Eastman 40, Eastman Speedway, and W & W Panchromatic plates respectively.*

Emulsion	Density of test object	Resolving power for different values of gamma				Mean value of resolving power
		$\gamma=0.31$	$\gamma=0.78$	$\gamma=1.06$	$\gamma=1.47$	
Eastman Speedway	0.16	10	10	10	10	10
	0.30	20	20	20	20	20
	0.60	35	35	25	30	31
	0.83	40	35	35	35	36
	1.17	45	35	38	40	40
	1.50	40	40	30	30	35
	2.07	55	42	40	40	44
	2.60	45	50	45	45	46
	3.05	55	42	45	43	46
	4.00	60	35	30	40	41

case, however, there must be one or more factors bearing inverse relationship to that of gamma and by an approximately equal amount.

If what we have just stated is true, namely, that the resolution decreases as the density increases where the increased density is gained by exposure, and that the decrease in resolving power is due to a spreading of the latent image because of scattered light in the emulsion, then,

TABLE 6. *The dependence of the resolving power on the density, D, of the opaque sections of the test object. Where D=log C and C is the contrast of the test object. The tables are for Eastman Process, Eastman 40, Eastman Speedway, and W & W Panchromatic plates respectively.*

Emulsion	Density of test object	Resolving power for different values of gamma				Mean value of resolving power
		$\gamma=0.49$	$\gamma=0.88$	$\gamma=1.18$	$\gamma=1.43$	
Wratten Panchromatic	0.16	15	15	15	10	14
	0.30	30	25	20	20	24
	0.60	40	40	25	25	32
	0.83	45	40	28	30	36
	1.17	55	50	35	30	42
	1.50	65	45	35	30	44
	2.07	50	—	35	30	45
	2.00	55	60	40	30	46
	3.05	57	57	35	25	43
	4.00	60	55	40	30	44

similarly, extended development simply causes a further growth of the developed image. In other words, scattered light produces a latent image whose size on development could have been increased either by developing a given exposure for a longer time, or by rendering the image more developable by a longer exposure and development for a given time. Briefly, the size of a developed photographic image photographed according to a given set of conditions can be increased either by exposure or by development.

Fig. 4 represents graphically what occurs under an opaque line element, E, placed in contact with the emulsion and exposed to collimated radiation in the direction mn. On account of the turbidity of the emulsion a certain fraction of the incident radiation will be scattered under the geometrical edge of E. The intensity of the scattered radiation will decrease approximately logarithmically from the edge.[4] Thus

[4] F.E. Ross. Photographic Sharpness and Resolving Power. Astro. Phys. J., *52:* p. 201; 1920.

the developed image will have a structure such that densities at successive points plotted against their respective distances from the geometrical edge of E will have the form of a characteristic curve, shifted into the geometrical shadow according to exposures shown by F. E. Ross.[5] There are two portions of the curve to be considered: namely, the under exposed portion called the toe, and the straight line portion. As already pointed out above, for very small exposures only the toe with a low density-exposure gradient is present and the resolution

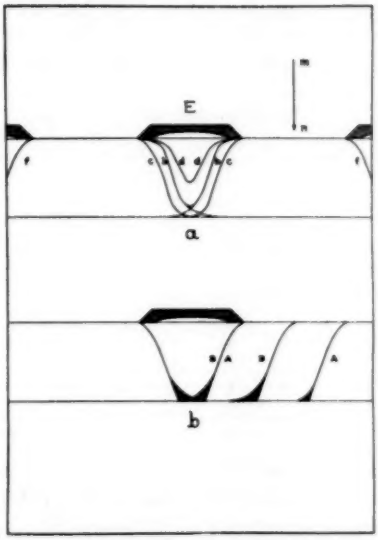


FIG. 4. *Sketch showing growth of density under the geometrical edge of the line element, E, with exposures for a given time of development.*

is low. Now as the exposure is increased the density commences to build up on the straight line portion of the curve as shown in Fig. 5, and the resolving power increases until the growth in density has reached a point where the toes of the two curves *bb* meet as shown in Fig. 4*a*. Whereupon the contrast between the two images immediately decreases with a corresponding decrease in the resolution. With additional exposure the contrast is reduced until the eye finally fails to pick up any contrast between *cbd*, and *dbc*, and the two lines are no longer resolved. Obviously, if a measuring device were used having a higher contrast

[5] F. E. Ross. Ibid.

sensibility than the eye, then the photographic material would have a higher resolving power.

Another case of the role played by the under exposed portion of the characteristic curve in this connection is illustrated by two emulsions *A* and *B* having approximately the same maximum resolving power, but *A* rendering a much cleaner image than *B* although the fog values, are equal. If the two emulsions have equal characteristic curves and

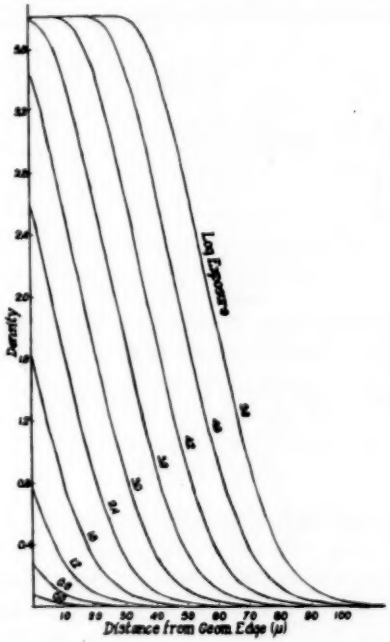


FIG. 5. *Diagram showing the growth of density, with exposure at different distances from the geometrical edge.*

equal scattering characteristics then for exposures producing equal densities, the definition point by point should be equally good. If however, *A* and *B* have the same straight characteristic and the same scattering characteristics, but the former has a smaller toe, it will have a cleaner image in the under-exposure region, Fig. 4*b*.

The relation between resolving power and contrast in the test object is shown in Table 1 and Fig. 6. These data are for the Eastman 33 emulsion. The values in the second column of Table 1 are the densities

of the test object, columns 3, 4, 5, and 6 contain the resolving power values obtained with the test object density listed on the same horizontal line in column 2. The gamma to which each series of exposures was developed is listed at the head of each column. The last column gives the arithmetical mean of the values in columns 3 to 6 inclusive. We will refer now to Fig. 6. This curve is plotted from the average values in column 7, Table 1. It is seen that the resolution changes very rapidly with contrast up to a density of about 0.6 when it starts to turn over approaching its maximum value at a density of about 2.5. This curve

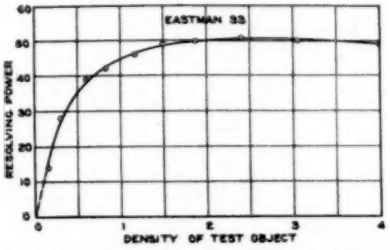


FIG. 6. *The curve shows the dependence of resolving power on the density, D, of the opaque sections of the test object for an Eastman 33 emulsion. Where D=log C and C is the contrast of the test object.*

is of the exponential type and is well represented by a formula of the form

$$R=C(1-e^{-\alpha D}) \tag{5}$$

where C and α are constants evaluated from values of R and D taken from the curve. The mean value of α calculated by formula (5) for the Eastman 33 was found to be 1.01; and the value of C which corresponds to the value of R, when D is infinite, is 51. Using these values of α and C the value of R was calculated for several values of D. These values are listed in column 5, Table 2, opposite the observed values in column 4 for the same densities. The agreement between the two columns is within the experimental error of observations. These data are represented graphically in Fig. 7.

No attempt has been made to establish the above formula on a theoretical basis; the data yet available are insufficient to make speculation profitable. It is quite evident, however, that since at low values of contrast, that is, at low test object densities, the ratio between the intensity of radiation transmitted through the clear lines and that transmitted through the opaque spaces and background is low, the

radiation acting to produce developability under the line element E, Fig. 4a, is no longer due solely to radiation scattered in the emulsion since a certain fraction of the incident radiation is transmitted through the opaque section. The lower the density the more radiation will be transmitted through the opaque section aiding in the production of an image under the line element E. At high values of contrast the ratio

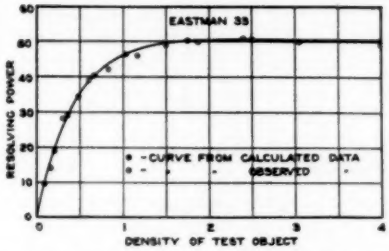


FIG. 7. *Gives a comparison between the observed values of the relation between contrast and resolving power and the values of the relation computed from formula (5).*

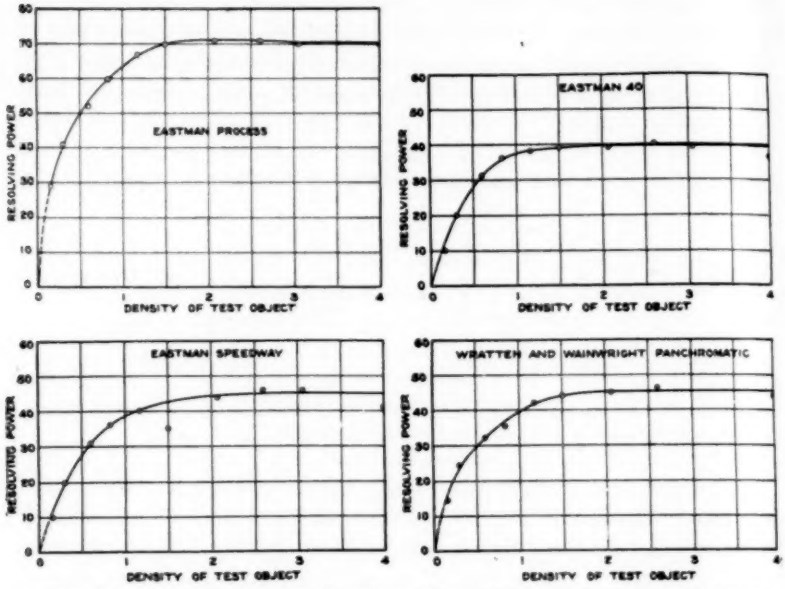
T_c/T_0 is very high and the image formed under the line E is due only to scattered radiation. Now, as the contrast decreases, the above ratio decreases and the resolution is diminished until in the limit as the ratio approaches unity the resolution approaches zero. One would expect the function to be of the exponential form since the change of T_c/T_0 with D is exponential.

TABLE 7. *A comparison of the averages of the observed values of R in the last column of Tables 3 to 6 and the corresponding values computed from formula (5).*

Emulsion	C	D	$R_{obs.}$	$R_{calc.}$	α
Eastman Process	71	0.058	10	9.7	1.14
		0.10	20	16.8	1.43
		0.18	30	27.5	1.31
		0.32	40	41.3	1.12
		0.51	50	53.2	1.04
		0.83	60	63.5	0.98
		1.50	70	66.3	1.26
		2.50	71	70.6	1.18
				Mean	1.18

Additional data are given for several other standard emulsions in Tables 3 to 6 inclusive. The mean values of the resolving powers, that

is, the data in the last column of each table, are shown graphically in Figs. 8 to 11 respectively. All these are of a similar form to that in



FIGS. 8–11. *These curves show the dependence of the resolving power on contrast in the test for Eastman Process, Eastman 40, Eastman Speedway, and W. & W. Panchromatic Plates.*

Table 2 and the curve in Fig. 7. Each is well represented by formula (5) as will be seen by referring to tables 7 to 10 inclusive. The calculated values of R in column 5 are in good agreement with the corresponding observed values of R in column 4.

TABLE 8. *A comparison of the averages of the observed values of R in the last column of Tables 3 to 6 and the corresponding values computed from formula (5).*

Emulsion	C	D	$R_{obs.}$	$R_{calc.}$	α
Eastman 40	40	0.12	10	10.8	1.02
		0.30	20	21.9	1.00
		0.57	30	31.1	1.05
		0.80	35	35.5	1.12
		1.50	39	37.3	1.07
		2.60	40	39.8	
				Mean	1.05

It may be interesting to consider what the difference would be if one used a test object with black lines on a transparent background. The essential difference would be a considerable increase in the stray radiadue to a higher total transmission of the test object. Due to internal reflection in the lens system and reflection from the back of the photographic plate a certain fraction of the transmitted radiation will be spread more or less uniformly over the image, the effect of which will

TABLE 9. *A comparison of the averages of the observed values of R in the last column of Tables 3 to 6 and the corresponding values computed from formula (5).*

Emulsion	C	D	$R_{obs.}$	$R_{calc.}$	α
Eastman	46	0.135	10	10	0.795
Speedway		0.31	20	19.8	0.797
		0.57	30	29.7	0.804
		0.77	35	36.1	0.810
		1.12	40	40	0.787
		2.10	45	42.9	0.762
		2.60	46	45.7	
				Mean	0.792

TABLE 10. *A comparison of the averages of the observed values of R in the last column of Tables 3 to 6 and the corresponding values computed from formula (5).*

Emulsion	C	D	$R_{obs.}$	$R_{calc.}$	α
Wratten	45	0.09	10	9.1	1.21
Panchromatic		0.22	20	19.3	1.16
		0.33	25	25.5	1.29
		0.51	30	32.6	0.94
		0.75	35	38.3	1.22
		1.06	40	41.6	0.90
		1.50	44	41.9	1.01
		2.60	45	44.8	
				Mean	1.10

be to lower the contrast. Hence for a test object of a given density the resolving power will be lowered by a certain amount depending on the quantity of stray radiation and the density of the test object.

SUMMARY

The dependence of resolving power on the contrast of the test object was determined for five standard photographic emulsions.

At low values of contrast the resolving power is low changing rapidly as the contrast increases. Being zero at contrast of unity it reaches

76 per cent of its maximum value at a test object density of 0.6, that is a contrast of 4 and 83 per cent at a density of 0.85 or contrast of 7.1. It very nearly reaches its maximum value at a density of 2 to 2.5 or a contrast of from 100 to 320.

The relation between density of test object and resolving power is expressed by the formula

$$R=C(1-e^{-aD})$$

where R is the resolving power and D is the density of the opaque sections of the test object.

Resolving power varies between wide limits with the exposure. It is found to be practically independent of gamma.

KODAK RESEARCH LABORATORIES,
ROCHESTER, N. Y.,
OCTOBER, 1927.

INSTRUMENT SECTION

LIGHT FILTERS FOR THE ISOLATION OF NARROW SPECTRAL REGIONS

By Loyd A. Jones

There are many problems in the fields of physics, chemistry, biology, physiology, etc., which require the isolation of fairly narrow spectral regions in order that the effects produced by radiation of different wavelengths may be determined. Obviously, the most elegant solution of the problem is the use of some type of dispersing instrument such as a monochromatic illuminator which will deliver radiation of high spectral purity. In many cases, however, it is not feasible to employ this method because of the high cost of such equipment or because of the impossibility of obtaining the required radiation intensity. For the isolation of relatively narrow spectral regions it is sometimes necessary therefore, to use light filters made up of selectively absorbing materials. For this purpose there are many materials available, among which may be mentioned solutions, glasses, dyed gelatin, etc. Obviously, the particular method and materials used will be governed largely by the requirements of the problem.

The filters described in this paper were developed at the request of Dr. Ethel M. Luce, who is investigating the action of radiation in the prevention and cure of rickets in white rats. That the results might be based on an average of a large number of observations and therefore be most reliable, this work involved the use of many of these animals. When a large number of these animals are to be subjected to radiation treatment, it is desirable that several be exposed at the same time. This necessitates the illumination of a relatively large area. Moreover it is desirable that the radiation intensity incident on the animals be relatively high so that the period of treatment need not be unduly prolonged. Since the treatment of a given group of animals may extend over a period of three or four months, it is obviously desirable that the filters be as stable as possible to avoid the necessity of constantly rechecking and making up new filters.

* Communication No. 335 from the Kodak Research Laboratories.

The possibility of using a monochromatic illuminator for this purpose was considered but it appeared that the requirements of area to be illuminated and radiation intensity desired could not be met without the construction of a very special instrument which would be almost prohibitive from the standpoint of cost. A careful survey of the selectively absorbing materials available was therefore made and the filters described in the following pages were constructed. The materials employed were as follows:

Distilled water, H_2O
Aqueous solution of copper sulfate crystals, $CuSO_4$, $5H_2O$
Aqueous solution of nickel sulfate crystals, $NiSO_4$, $7H_2O$
Glass, Corning No. G986A
Glass, Corning No. G586
Glass, Corning No. G980A
Dyed gelatin, Wratten filter No. 88A
Dyed gelatin, Wratten filter No. 25
Dyed gelatin, Wratten filter No. 61
Dyed gelatin, Wratten filter No. 49

ABSORPTION CHARACTERISTICS OF MATERIALS

It is well known that water absorbs very strongly in the infrared region and hence may be used to absorb radiation of long wave length. In Fig. 1 curve *C* shows the relation between density and wave length, density *D* being defined in the usual manner as the logarithm of the reciprocal of the transmission. Curve *C* is plotted from data published by Coblentz[1] and applies to a layer of water 2 cm thick. With respect to the absorption of water in the infrared region, Coblentz states that "Water is the most opaque substance known, a layer 1 cm thick absorbing all wave lengths greater than 1.4μ."

With the exception of curve *C*, all the spectrophotometric data shown in Figs. 1, 2, 3, and 4 were obtained in this laboratory. In order to cover the wave length range from 200 mμ to 1400μ four different spectrophotometers were used:

(a) A quartz spectrograph, made by Adam Hilger, with the Hilger spectrophotometric attachment. This covers the wave length region from 200 to 600 mμ.

(b) A spectrophotometer of the Hüfner type made also by Adam Hilger. This is a visual instrument covering the wave length range from 400t o 700 mμ.

[1] W. W. Coblentz, Bull. Bur. Stand., 7, p. 660; 1911.

(c) A photographic spectrophotometer covering the wave length range from 600 to 900 mμ. This consists of a multiple prism instrument made by Hans Heele and used with photographic plates sensitized for the wave length region mentioned.

(d) An infrared spectrophotometer made by Adam Hilger using a thermopile as sensitive element. This was used in the region of wave length greater than 800 mμ.

In Fig. 1 the curve *A* shows the absorption characteristics of a 5 per cent solution of copper sulfate having a thickness of 2 cm. The curves as plotted refer specifically to the absorption of the solution itself, a correction having been made for reflection losses at the quartz-air surfaces of the cell in which the solution was held during measurement. It will be noted that this solution transmits freely wave lengths

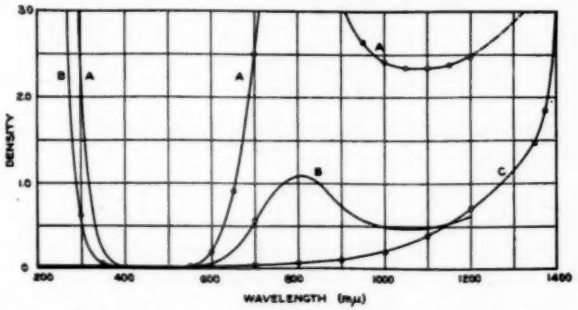


FIG. 1. *Curve A 10%—aqueous solution $CuSO_4$, $5H_2O$—1 cm. Curve B 2%—aqueous solution $CuSO_4$, $5H_2O$—1 cm. Curve C H_2O—2 cm.*

between 320 and 650 mμ. A second transmission band is found in the region between 920 and 1250 mμ with the maximum at approximately 1080mμ. The transmission in this region, however, is not great, the maximum being approximately .4 per cent ($D = 2.36$). It is evident that this solution is very useful for absorbing radiation of wave length greater than 650. In the concentration and thickness as shown by curve *A*, the absorption at 300 mμ is relatively great and hence, if it is desired to transmit the radiation of 300, it is necessary to dilute the copper sulfate solution or to dispense with it entirely. Curve *B* shows the absorption for a 1 per cent solution copper sulphate in a thickness of 2 cm. Its density at 300 mμ is .6 corresponding to a transmission of 25 per cent. At this dilution, however, the absorption in the infrared region has decreased markedly and hence the solution is not of great value for the removal of the long wave radiation.

In Fig. 2 is shown the spectrophotometric curve for a 50 per cent solution of nickel sulfate of thickness 1 cm. This material shows three rather sharp transmission bands having maxima at 300 mμ, 530 mμ and 860 mμ. The transmission at the maxima of the bands in the ultraviolet and visible regions is very high. This material has been found

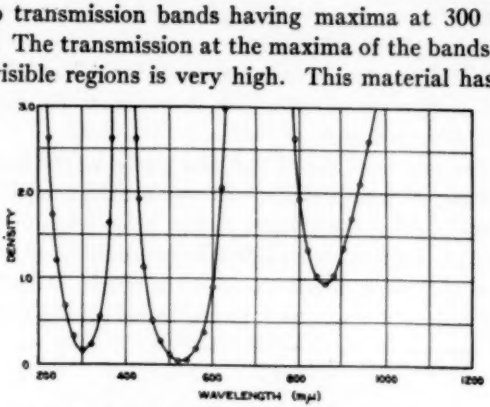


FIG. 2. *50%—aqueous solution $NiSO_4$, $7H_2O$—1 cm.*

useful in combination with other absorbing elements for the isolation of narrow regions.

In Fig. 3, the spectrophotometric curves for Corning glass No. G986A are given. Curve *A* is for a layer 5 mm thick, while curve *B* applies to a thickness of 3.2 mm. This shows two very marked trans-

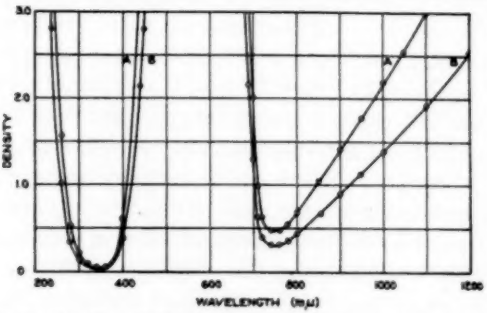


FIG. 3. *Curve A Corning Glass G 986A—.50 cm. Curve B Corning Glass G 986A—.32 cm.*

mission bands, one in the ultraviolet and one in the near infrared. The absorption of this material for radiation greater than 1200 is rather unique. The author has examined but few colored glasses which absorb appreciably in this region.

In Fig. 4 are given curves applying to Corning glass G586. Curve *A* is for a layer 5 mm thick and curve *B* for a thickness of 3.2 mm. The

curve for the 3.2 mm thickness in the ultraviolet is not shown as it lies so close to the curve for 5 mm as to make the plotting very inconvenient. This material also absorbs to an appreciable extent in the region between 800 and 1060 mμ but for longer wave lengths its absorption decreases rapidly. The transmission band in the ultraviolet is somewhat narrower than in the case of G986A. This glass may therefore be utilized to eliminate the radiation of wave length 300, while the G986A transmits that wave length rather freely.

In Fig. 5 curve *E* shows the absorption of the Wratten filter No. 88A. It is practically opaque between the region 200 and 720 mμ and very transparent for all wave lengths longer than 720 mμ. The correction for loss by surface reflection has not been applied to this curve and the

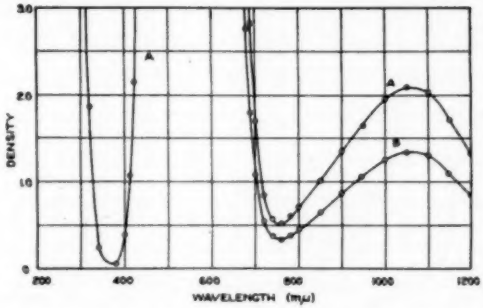


FIG. 4. *Curve A Corning Glass G 586—.50 cm. Curve B Corning Glass G 586—.32 cm.*

density in the region between 800 and 1200 mμ, 0.04, is largely attributable to the loss by surface reflection. It is evident therefore that this filter transmits very freely in this region. As a matter of fact a measurement made with this filter shows that it is absorbing only 16 per cent of the total radiation from a tungsten filament operating at approximately 2800°K. Of this 16 per cent, 8 per cent (approximately) is attributable to loss by surface reflection, and the greater part of the remaining 8 per cent is attributable to absorption in the region of wave length shorter than 720 mμ. This filter may therefore be considered as highly transparent for all wave lengths longer than 720 mμ.

Curve *D* (Fig. 5) shows the absorption of the Wratten filter No. 25. It cuts sharply at wave length 590 mμ and is very transparent in the region of longer wave lengths. Although the complete curve is not shown, it is known that its transmission throughout the region between 700 and 1200 mμ is very high.

Curve *C* is for Wratten filter No. 61 which transmits a relatively narrow band between wave lengths 490 and 590 mμ having a maximum transmission at 540 mμ. This filter also transmits the entire infrared region between 800 and 1200 mμ.

Curve *B*, for Wratten filter No. 49, is somewhat asymmetrical, having a maximum transmission at 460 mμ and embracing the wave length band from 380 to 490 mμ. It also transmits freely in the infrared region, 750 to 1400 mμ.

The statements which have been made relative to the cut-off or transmission band of these filters are of course qualitative and are based upon the wavelength at which the transmission becomes 1 per cent ($D=2.0$). The curves in Fig. 5 (*B*, *C*, *D*, and *E*) refer to gelatin filters and have not been corrected for loss by surface reflection.

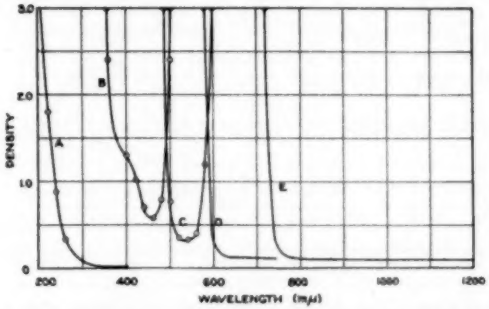


FIG. 5. *Curve A Corning Glass G 980A—.50 cm. Curve B Wratten filter No. 49, Curve C Wratten filter No. 61, Curve D Wratten filter No. 25, Curve E Wratten filter No. 88A.*

Curve *A*, Fig. 5, is for Corning glass G980A, 5.0 mm thick. This glass is very transparent in the ultraviolet, visible, and infrared regions. The curve as shown has been corrected for loss by surface reflection and refers to the absorption characteristics of the glass itself. At wave length 300 it has a density of .09 corresponding to a transmission of 80 per cent. In the region between 320 and 1200 its transmission is extremely high. This glass absorbs only 12 per cent of the radiation emitted by a tungsten filament operating at 2800°K of which 8 per cent is attributable to loss by surface reflection. For many purposes it is a satisfactory substitute for quartz in that it is transmitting freely in the region around 300, at which ordinary glass is quite opaque. It is particularly useful in making up liquid filter cells when it is necessary to use a cell having two walls and when it is desired to keep the total thickness of a colored glass to a minimum. For example in some cases

it is found that 2 mm of G986A is sufficient to give the required absorption. If this is divided into two cell walls the thickness is only 1 mm which is insufficient for mechanical strength. By using the entire 2 mm of G986A as a single plate for one cell wall and a plate of G980A 2 or 3 mm thick a cell of satisfactory strength is obtained.

ABSORPTION CHARACTERISTICS OF THE FILTERS

Filter No. 1, entire infrared region. For this purpose Wratten filter No. 88A used alone is quite satisfactory. As shown by the curve in Fig. 5, it transmits freely all wave lengths greater than 720 mμ, transmitting 84 per cent of the total radiation emitted by a tungsten filament at 2800°K.

Filter No. 2, partial infrared. By combining Wratten filter No. 88A with a 2 cm layer of water, a filter is obtained having the characteristics shown in Fig. 6, transmitting between 720 and 1380 mμ. Its maximum

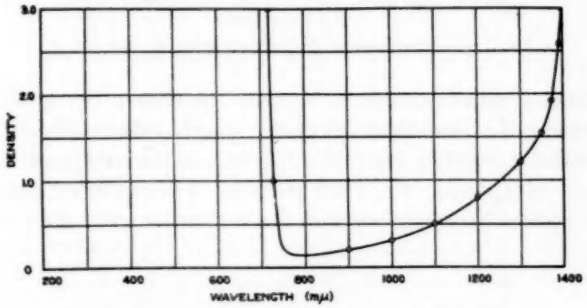


FIG. 6. *Spectrophotometric absorption curve for filter No. 2.*

is at approximately 800 mμ at which wave length it has a density of .14 corresponding to a transmission of 72 per cent. The curve as plotted is the characteristic for the combination taking into account the loss by surface reflection.

Filter No. 3, partial infrared. This filter is made up by using Wratten filter No. 88A with 2 cm water and 3.2 mm Corning glass G986A. Its absorption characteristic is shown by the curve in Fig. 7. The transmission band is considerably narrower than for filter No. 2, extending from 720 to 1020 mμ. The maximum occurs at wave length 770 where the density is .5 corresponding to a transmission of 35 per cent.

Filter No. 4, orange red. This filter is composed of Wratten filter No. 25 combined with a 2 cm 5 per cent aqueous solution of copper sulfate crystals. The absorption characteristic is shown in Fig. 8. The trans-

mission band lies between wave lengths 590 and 690 with a maximum at 630 mμ at which wave length the density is 0.62 corresponding to a transmission of 26 per cent. It will be noted that there is another transmission band having a maximum at 1080 mμ. The maximum transmission however is relatively low, being only 0.4 per cent ($D=2.36$).

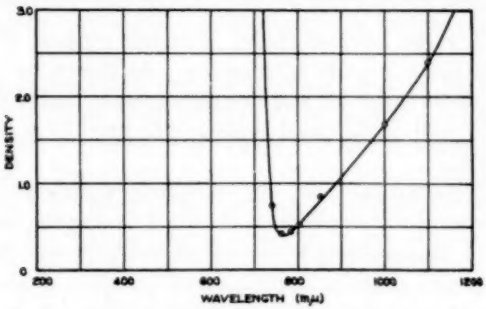


FIG. 7. *Spectrophotometric absorption curve for filter No. 3.*

Of course it is quite possible to increase the absorption in this region by increasing the concentration of the copper sulfate solution. This, however, has a disadvantage that it decreases the maximum transmission in the orange red. For most purposes it seems more advisable to tolerate the small transmission in the region between 1000 and 1200

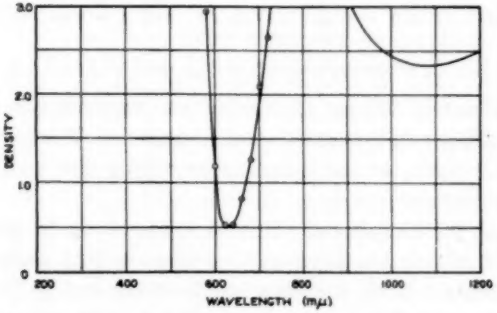


FIG. 8. *Spectrophotometric absorption curve for filter No. 4.*

rather than to decrease the transmission at the desirable wavelength. The requirements of the particular case, however, may determine whether or not it is advisable to increase or decrease the concentration of the copper sulfate used for the orange red region.

Filter No. 5, green. This filter is made by combining Wratten filter No. 61 with a 2 cm layer of a 5 per cent copper sulfate solution. Its absorption characteristics are shown in Fig. 9. The filter transmits between wave lengths 490 and 600 mμ having a maximum at 530 at which wave length the density is 0.28 corresponding to a transmission of 52 per cent. If desired the transmission between 1000 and 1200 mμ

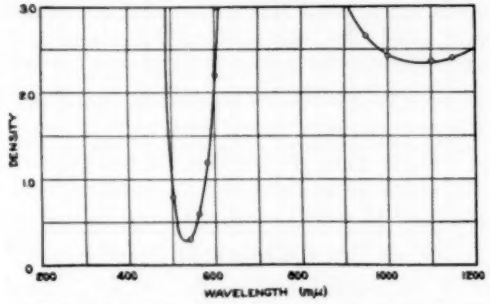


FIG. 9. *Spectrophotometric absorption curve for filter No. 5.*

can be decreased to less than .1 per cent by increasing the concentration or by using a 3 cm layer of the 5 per cent copper sulfate solution instead of 2 cm. This will not appreciably affect the maximum transmission in the green region.

Filter No. 6, blue. This is made by combining Wratten filter No 49 with a 2 cm layer of 5 per cent copper sulfate solution. The absorp-

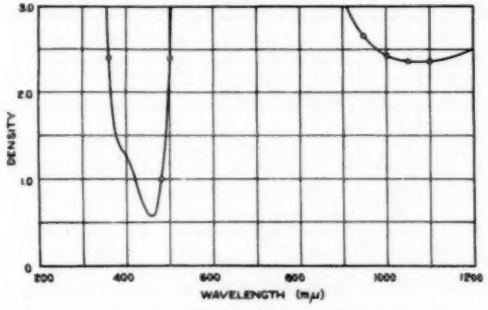


FIG. 10. *Spectrophotometric absorption curve for filter No. 6.*

tion characteristic is shown in Fig. 10. The transmission band lies between 380 and 500 mμ with a maximum at 460 mμ at which wave length the density is .58 corresponding to a transmission of 26 per cent. Here again it is possible to eliminate the transmission band shown be-

tween 1000 and 1200 by increasing the concentration of the copper sulfate. Such procedure will not appreciably decrease the transmission in the blue region.

Filter No. 7, near ultraviolet. This filter is made by combining 2 cm of 10 per cent copper sulfate solution with 3.2 mm Corning glass

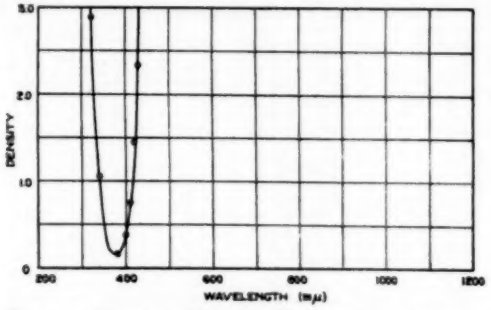


FIG. 11. *Spectrophotometric absorption curve for filter No. 7.*

G586. The transmission characteristic is shown in Fig. 11. The filter transmits between 330 and 430 mμ with a maximum at 380 mμ at which wave length the density is .16 corresponding to a transmission of 69 per cent. The combined absorption of the glass and the 10 per cent copper sulfate solution reduces the transmission at all wave lengths between 430 and 1200 mμ to less than 0.1 per cent.

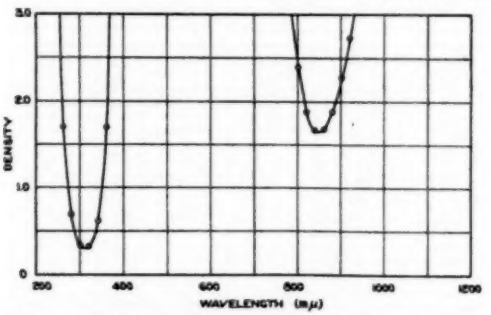


FIG. 12. *Spectrophotometric absorption curve for filter No. 8.*

Filter No. 8, ultraviolet. This is made up by using a 1 cm layer of a 50 per cent aqueous solution of nickel sulfate crystal combined with 3.2 mm of Corning glass G986A. The absorption characteristic is shown in Fig. 12. Transmission is between 260 mμ and 360 mμ, with a maximum at 310 mμ, at which wave length the density is 0.3 correspond-

ing to a transmission of 50 per cent. With this combination there is a second transmission band having a maximum at 850 mμ. The maximum transmission, however, is relatively low, being 2.2 per cent ($D=1.65$). By using with the components already specified, a 1 cm layer of 2 per cent aqueous solution copper sulfate crystals the transmission band shown in Fig. 8 at 830 mμ can be practically eliminated. Reference to Fig. 1 shows that copper sulfate has a maximum absorption at approximately 810 mμ and a 2 per cent solution of this material used in a 1 cm layer will reduce the transmission of filter No. 8 in the infrared to less than 0.1 per cent at all points. The use of copper sulfate in this filter, however, has its objections, since it will be seen by again referring to Fig. 1 the copper sulfate absorbs strongly at 300 mμ, the result being that the transmission on the short wavelength side of the transmission band is materially decreased. If it is desired, however, to obtain a filter having a narrower transmission band than that shown in Fig. 12 a copper sulfate solution can be used to advantage. By this means all wave lengths shorter than 310 may be absorbed, thus narrowing the transmission band but of course decreasing the transmission at the maximum.

In Table 1 is given a summary showing the composition and characteristics of these filters. The author realizes that these filters are far from ideal but they seem to be about the best that can be obtained with the materials available and to meet the requirements of the particular problem for which they were developed. The stability of the dyed gelatin filters is excellent although of course if they are subjected to extremely high illuminations and temperatures some change may occur. The stability of the solutions is also entirely satisfactory. No specific tests of the stability of the colored glasses have been made but there is some indication that the transmission of the G980A glass at wave length 300 decreases somewhat after exposure to relatively high intensities of ultraviolet radiation. The spectrophotometric data given in this paper relative to the colored glass apply to the particular samples which were used in this work. The difficulty in manufacturing glass which maintains constant absorption characteristics from batch to batch is well known. It will probably be necessary, therefore, for anyone attempting to make up duplicates of these filters, to make spectrophotometric measurements on the particular pieces of glass used.

It is probable that somewhat different combinations of the components will yield filters more suitable for other purposes. The filters

for splitting the infrared region are particularly inadequate but so far as the author is aware, no better materials are available. From the standpoint of the anti-rachitic problem, filters No. 7 and 8 are particularly interesting. No. 7 transmits practically no radiation of wave length less than 320, while No. 8 transmits between 280 and 320 rather freely. Experiments with these two filters should therefore answer with some certainty the question as to whether radiation of wave length longer than 320 is effective in the treatment of rickets.

One or two of these filters described are not precisely identical with those used by Dr. Luce in her work, slight modifications having been

TABLE 1. *Summary of components and characteristics of filters.*

No.	Material	Concentration	Thickness	Wave length limits	Wave length max.	T_{max}
1	88A	—	—	720–λ>1400	720–λ>1400	.80
2	88A H_2O	— —	— 2. cm	720–1380	800	.72
3	88A G986A	— —	— .32 cm	720–1020	770	.35
4	No. 25 $CuSO_4, 5H_2O$	— 5%	00 2. cm	590–690	630	.26
5	No. 61 $CuSO_4, 5H_2O$	— 5%	— 2. cm	490–690	530	.52
6	No. 49 $CuSO_4, 5H_2O$	— 5%	— 2. cm	380–500	460	.26
7	G586 $CuSO_4, 5H_2O$	— 10%	.32 cm 2.0 cm	330–430	380	.69
8	G986A $NiSO_4, 7H_2O$	— 50%	.32 cm 1.0 cm	260–360	310	.50

made in composition in order to fulfill more closely the ideal requirements of her particular problem. In the opinion of the author the formulas as given in this paper result in a series of filters of more general usefulness.

The cells for holding the liquid components are made in the form of shallow trays approximately 11 inches wide, 18 inches long, 2 inches deep. In the bottom of each tray are six rectangular openings each

approximately 4½ inches wide and 5½ inches long. These openings are filled by plates made of the glass required to give the desired spectral absorption. A liquid tight joint is obtained by the use of gaskets, the glass plates and gaskets being drawn firmly into position by using small metal frames screwed to the bottom of the tray from the under side. The trays are made of cast brass heavily silver plated to prevent attack by the solutions used. Considerable difficulty was experienced in finding a gasket material which would maintain a liquid tight joint over long periods during which the trays were alternately heated to relatively high temperatures during the exposure to the radiation and cooled between periods of use. The best material for making these gaskets was found to be pure unvulcanized gum rubber sheets. This material is very soft and as the plates are pulled into position with the screws the glass embeds itself in the rubber and the heating of the trays then seems partially to vulcanize the rubber and forms a durable and liquid tight joint. The bottom of the trays appears very much like a window frame with two transverse and one longitudinal stile. These stiles are made as narrow as possible so as to offer little obstruction to the transmission of radiation. The total area of glass is approximately 150 square inches. The area which may be illuminated is 10 inches wide and 16 inches long. The tray is placed over the area to be illuminated and the light source suspended directly above the center of the tray. The gelatin filters are placed below the tray and hence are protected to a certain extent from the direct radiation of the light source. The liquids used in the tray, since they absorb the long wave radiation to some extent serve also to protect the glass from over heating. Four filter trays were made up, one being filled with sheets of clear white optical glass, one with G986A, one with G586, and the fourth with G980A.

The spectrophotometric measurements relative to these filters were made by Mr. A. L. Schoen and Mr. E. E. Richardson of this laboratory. The author wishes to express his appreciation to them for this very material assistance involving the expenditure of much time and effort.

KODAK RESEARCH LABORATORIES,
ROCHESTER, N. Y.,
OCTOBER, 1927

A SUGGESTED ARRANGEMENT FOR THE DETERMINATION OF THE VELOCITY OF LIGHT

By Noel Deisch

In the method most commonly used for the determination of the velocity of light a series of regular interruptions or pulsations is superposed on a collimated beam, and the retardation suffered by the beam in its passage to a distant station and return, which discloses itself in stroboscopic coincidences or phase effects, is taken as a direct measure of the velocity. With a base of given length, and established with a given probable error, the accuracy that may be attained is determined (1) by the order of frequency of the modulated beam (2) by the precision that can be reached in controlling and in measuring this frequency, and (3) by the closeness with which the phase relation of the incoming and the outgoing beam can be detected and adjusted.[1]

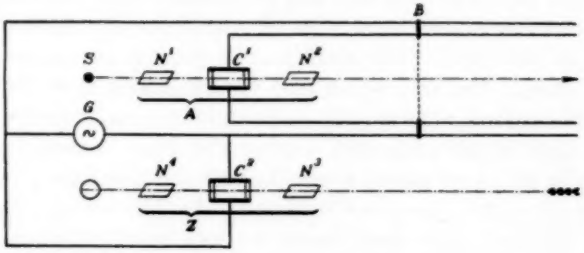


Fig. 1. *Beam modulating apparatus.*

In the present method it is sought to avoid the limitations inherent to the use of a mechanical device such as a rotating multi-sectored disk or multi-faced mirror for interrupting or modulating the beam by employing an electro-optic modulator, or preferably a pair of electro-optic modulators coordinately energized from the same oscillating circuit. An arrangement of this latter type is shown in Fig. 1, in which the generator of the pulsating beam is represented by the assembly A, and the observing component of the instrument, through which the phase relation of the pulsations in the incoming and the outgoing beam is determined, is represented by the assembly Z. A and Z are similar,

[1] The theory relating these and other factors was analytically treated in great detail by Cornu in his elaborate memoir published in the *Annales de l'Observatoire de Paris; Memoires, 13*, 1876.

and represent conventional groupings, C^1 and C^2 being electro-optic cells functioning between polarizers N^1 and N^3, respectively, and analyzers N^2 and N^4. N^1 is turned on the common axis of A through 45° with respect to C^1 and N^2. N^3 is oriented in the same plane as N^2, whereas C^2 and N^4 are turned on the common axis of the system Z through 45° with respect to N^3.

The electro-optic cells are supplied with a high-frequency oscillating potential by the oscillation generator G, the constants of the circuit being so arranged that a degree of birefringence in C^1 and C^2 is set up sufficient to produce at opposed crests of the electric oscillations retardations resulting at the one crest in complete transmission and at the other crest in complete extinction of monochromatic light. Under these conditions the beam of light having its origin in the source S and emergent from N^2 will periodically vary in intensity from zero to a maximum, the frequency of this variation being of course coincident with the frequency of the oscillating circuit.

On entering Z after reflection at the distant station (not shown in the figure) the polarized and pulsating beam has a new series of pulsations superposed on it, the phase relation of which to the original series of pulsations will determine the transmission of the return beam by Z. In other words the degree of extinction in N^4 will be determined by the phase relation existing between the pulses of electric stress in the cell and the pulses of light in the beam. Other conditions being equal this phase relation will, with any given frequency, be settled by the lag of the return beam. Since the oscillating electric stress active in either cell may by well known methods, as by the sliding bridge B, be given a definite lead or lag over the stress active in the other cell, the phase condition just referred to can be brought to parity and the desired time relation established between the outgoing and the return beam. Due to the fact that the modulations are not as abrupt in contour as those produced by means of a rotating mirror, it will be necessary to use a photometric method in making these settings, analogous to the various photometric methods developed in connection with experiments with the toothed wheel.

The outstanding advantage of the arrangement outlined is that the pulsating beam can be simply generated and its frequency rigorously controlled through the use of a calibrated piezo-electric resonator. Moreover, very much higher frequencies can be had than are to be obtained with mechanical modulation methods. It must also be noted that the Tyndall glow along the path of the beam, which is often a

cause of considerable annoyance with existing methods, would be materially reduced at Z, for only a part of the light depolarized in being diffused would pass N^3.

The chief difficulty likely to be encountered in seeking to apply the proposal would appear to lie in securing a return beam of ample brilliance, due both to the inherently low transmission of the modulating system A and the necessity or at least desirability of working with only a comparatively narrow region of the spectrum. A truly monochromatic source would of course be ideal, the line 436 in the spectrum of mercury, which can be separated without recourse to the spectroscope, being naturally attractive in this connection. The case of an oscillating spark source, working in conjunction with the modulating system, is also interesting.

As a further practical note, it may be remarked that the electro-optic cells need not necessarily form part of the oscillating circuit proper (in which position their capacity would be a factor in lowering

FIG. 2. *Optical system of transmitter.*

the upper limit of attainable frequency) but could readily be put in a derivative circuit. This would allow of the transmitting component A and the receiving component Z of the apparatus being separated by some distance, thus optically isolating the outgoing and the incoming beams, and eliminating annoyance due to stray light and to scatter. The electrical capacity of the cells could moreover be reduced, and incidentally their "aperture" increased, by adopting the Karolus-Telefunken arrangement, wherein the electrodes are separated to a convenient distance and the sensitivity of the cell increased by giving it a considerable electrical bias. To reduce the dimensions of the nicols and of the electric-optic cells to a convenient value, the transmitter might be arranged somewhat as shown in Fig. 2, where S is a source, F a filter, L^1 a condenser, P a fore-polarizer, L^2 a collimator of short focus, A the modulator, L^3 a collecting lens, and L^4 a collimator of long focus.

When account is taken of the almost instantaneous recovery of certain dielectrics showing the Kerr effect, it would appear that the upper limit of frequency attainable by the use of an electro-optic modulator

should be fixed only by the capacity and the inductance of the circuit of which it forms a part. With the radio amateurs using frequencies well in excess of 50,000,000 cycles, the prospect of obtaining a pulsating beam of a frequency that would allow of close readings should be very good indeed, the above figure exceeding by a factor of at least 100 the best that can be done with rotating mirrors. It is an interesting fact that, with frequencies of the above order, corresponding to wave-lengths of six meters, the phase balance in the observing instrument Z might actually be carried out by lengthening or shortening the path of the beam instead of by the use of an electrical retarding device, an arrangement being used such as shown in Fig. 3, where L^1 is a collecting lens, L^2 a collimator, P^1 a totally reflecting prism movable along the

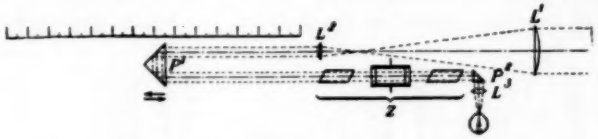


FIG. 3. *Diagram showing method of varying length of light-path.*

axis of the beam, which it returns on a parallel path, Z the receiving component proper, P^2 a deflecting prism, and L^3 the eyepiece.

Though a beam of continuously variable frequency, giving a succession of stroboscopic maxima and minima in the observing instrument similar to those obtained by the use of a variable speed rotating sector could be very easily produced by the method here described, a fixed frequency or a series of fixed frequencies (which might be the several higher harmonics of an oscillating circuit controlled by a piezo-electric crystal) would be preferred where the base is accurately known, since it would allow of closer or at least more conveniently obtained determinations of frequency. Where the distance is the variable factor, however, as it would be in surveying by the light-velocity method as suggested by Michelson, the use of a modulator continuously variable through its entire range of frequencies would be preferred.

WASHINGTON, D. C.
Dec. 10, 1927.

AN IMPROVED QUARTZ FIBER MANOMETER*

By Arnold O. Beckman

The quartz fiber manometer described by Haber and Kerschbaum[1] consisted of a single quartz fiber which was set into vibration by skillful tapping. Coolidge[2] substituted a double fiber for the single fiber, simplifying the attainment of plane vibration.

Recently in the use of a quartz fiber gauge trouble from room vibration was experienced. This was largely overcome by rigidly mounting the gauge in a heavy metal block. A convenient mounting is shown in Fig. 1. The gauge is inserted in the hole in the metal block, a layer of asbestos packed at the bottom, the whole heated above the

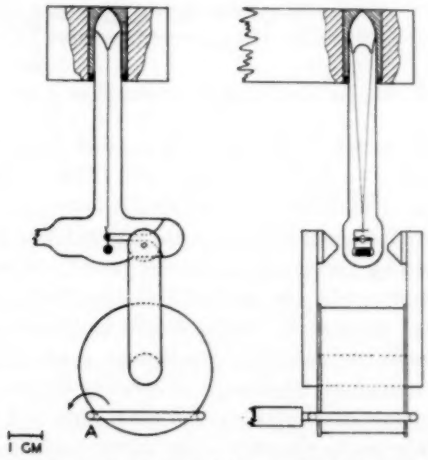


FIG. 1. *Two views of the starting device and the mounting of the quartz-fiber manometer.*

melting point of solder and the space filled with molten solder which is then allowed to cool. This provides a rigid mounting which can be heated if desired. A telescope with micrometer eyepiece for observing the amplitude of vibration is firmly attached to the same mounting,

* Contribution from Gates Chemical Laboratory, California Institute of Technology, No. 168.

[1] Haber and Kerschbaum, ZS. f. Elektrochem., *20*, p. 296; 1914.

[2] Coolidge, J. Am. Chem. Soc., *45*, p. 1637; 1923.

eliminating the trouble of shifting zero-point. The fiber is illuminated in the usual manner.

To facilitate setting the fiber in motion, the magnetic starting device shown in the figures was made. A glass-encased iron bob is suspended by two platinum wires from a horizontal wire about which the bob can rotate. The bearings on the supporting wires are easily made by fusing the wires into beads which are then flattened and drilled. One of the supporting wires extends above the axis and is so bent that it does not quite touch the quartz fiber when the bob is in its rest position (Fig. 2*a*) but engages it when the bob is moved slightly in a clockwise direction. As the bob is moved further, the bent wire slides beneath the end of the fiber and the fiber is thus released. (In case the presence of metal is undesirable, an all-glass mechanism could be simply made by attaching the bob to a glass wedge which pivots in a U-shaped

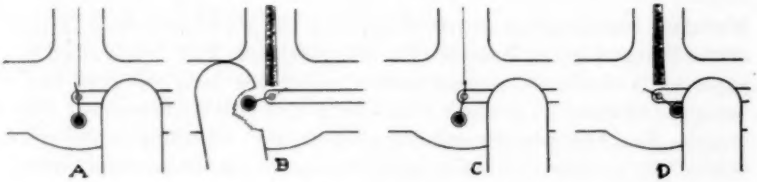


FIG. 2. *Steps in the operation of the starting device.*

supporting arm.) The bob is moved by the electromagnet which is conveniently placed in a rectangular frame so that it can be rotated about the axis *A* (Fig. 1).

The fiber is set into vibration in the following manner. With the circuit open, the normal positions of the bob and electromagnet are shown in Fig. 2*A*. The circuit is closed and the magnet is turned about the axis A until the bob has swung in the clockwise direction and the fiber is released (Fig. 2*B*). The magnet is returned until the bent wire just touches the fiber (on the right-hand side). The circuit is opened and the magnet is returned to its normal position (Fig. 2*C*). The device is now set. When the circuit is next closed, the bob swings in the counter-clockwise direction setting the fiber into vibration (Fig. 2*D*).

Two advantages result from bending the extended supporting wire so that, in its rest position, it does not quite touch the quartz fiber. One is that the fiber is suspended freely for the determination of the zero-point. The other is that when the circuit is opened and the bob falls to its rest position from the position shown in Fig. 2*D* the fiber is

immediately brought to rest due to the mutual damping effect of the vibrating fiber and the swinging bob.

The use of a rigid mounting and an internal starting device for a quartz fiber manometer is attended by the following desirable features:

1. The effect of external vibration is greatly reduced;
2. The zero-point does not change;
3. The fiber is quickly and positively set into vibration;
4. The fiber can be instantly brought to rest;
5. The initial amplitude of vibration is always the same, facilitating the reading of short time intervals and eliminating long waits on long time intervals.

GATES CHEMICAL LABORATORY,
CALIFORNIA INSTITUTE OF TECHNOLOGY,
PASADENA, CALIFORNIA. JAN. 12, 1928.

Metered combustion control as used at the Manchester Street Station of the United Electric Railways Co. of Providence, R. I. is described in a paper by A. S. Davis, Superintendent, which has been reprinted and is being distributed by the Leeds & Northrup Co., 4901 Stenton Ave., Philadelphia, Pa. The rate of combustion is regulated according to the steam demand by a master controller connected to the steam header so that it responds to drop in pressure due to flow of steam and sets up an electrical control current proportional thereto. This current cooperates with control devices which actually meter the pulverized coal and the air supplied to burn it to hold the two in a definite ratio and in proper amount to meet the steam demand. It is stated that the use of this equipment has resulted in more uniform steam pressure, and hence in more efficient operation of turbines, also in more uniform furnace conditions with lower excess air, resulting in higher boiler efficiency and lower furnace maintenance. Finally, the operators have been relieved of practically all of the physical work of operating the boiler, with the result that they are now able to pay closer attention to the finer details and to the better operation of the plant as a whole. Charts from recording instruments show a remarkably constant percentage of excess air and a uniform steam pressure, in spite of a rapidly fluctuating steam flow incidental to load swings in a traction plant. The percentage of CO_2 is set to be maintained as high as is compatible with economical operation of the furnaces, taking into consideration maintenance, combustible in ash and combustible loss up the stack, as well as heat lost in flue gases.

A NEW TYPE OF SELENIUM CELL

By R. Earl Martin

During the course of some experiments on selenium a need arose for a cell, the conducting part of which should be cylindrical. It was also necessary that the cell be of high sensitivity and consequently of high resistance.

SHAPING THE SELENIUM

Various methods for securing a uniform cylindrical piece of selenium of small cross sectional area were tried. It could not be cast by using glass or quartz tubing as a mould, for on cooling the continuity of the selenium was broken at a great many places along the rod. As it has a larger coefficient of expansion than these substances and the force of adhesion is larger than that of cohesion. It could not be turned down in a lathe to the proper size because of its brittleness. Finally the following very simple method was used.

Powdered vitreous selenium was placed in a pyrex test tube and heated in an electric furnace to a temperature slightly above its melting point, 217°C. The tube was removed from the furnace and the selenium poured slowly out of it. As the temperature of the selenium decreased it became more and more plastic. Drops would form on the edge of the tube; and then by turning the tube almost upside down the drops could be made to fall from its edge drawing out beautiful cylindrical threads of the viscous selenium of various diameters depending upon the temperature. The diameters of these threads depended on the temperature of the selenium, being smaller, the warmer it was. These threads, having a large exposed area, cooled rapidly and solidified into the desired cylinders of selenium.

ANNEALING OF THE SELENIUM

While much has been published[1] concerning the annealing of selenium cells it seems to me that a short discussion of the method employed is desirable.

To form a suitable electric heater a piece of coiled nichrome wire was wound on the outer surface of a funnel shaped piece of porcelain which was fitted into a lighting socket. The two ends of the wire were attached to the contact points of the socket. The resistance of the nichrome wire was about 40 ohms. This heater was placed in series with

[1] Dietrich, Phys. Rev., 2, *4*, p. 467; 1914. Bidwell, Phil. Mag., 5, *20*, p. 178; 1885. Also Beibl. Ann. Physik, *9*, p. 674; 1885. Brown, Proc. Iowa Acad. Sci., *23*, p. 241; 1916. Martin, Proc. Ind. Acad. Sci., *36*, p. 163; 1926.

a variable resistance, an ammeter and a 120 volt storage battery. A piece of copper plate about 0.3 cm. thick was placed over the open end of the heating element, and on top of this plate was placed the insulating form which was to hold the selenium. This form was either of plate glass or sheet mica.

A thread of the proper length and diameter was placed on this form. The threads at this stage are vitreous and resemble black glass in appearance. By gradually raising their temperture to about 130°C, they were transformed into the crystalline form of selenium which has a dull gray color. The temperature of the selenium was noted by a rather sensitive mercury-in-glass thermometer which was laid with its bulb on the insulating form which supported the specimen. After the thread has entirely crystallized the temperature was cautiously increased up toward its melting point. Meanwhile the surface of the thread near the place where it touched the insulating form was watched through a small magnifying glass so that the first sign of melting could be noticed. When this was observed, the temperature was quickly lowered about 6 or 7 degrees and held constant for 30 minutes. After this the temperature was lowered gradually to about 180°C and held there for several hours. Another 30 minutes was then spent in lowering the temperature to that of the laboratory. The cells were then placed in an air tight glass container which was thoroughly clean and dry and then kept in the dark for at least 24 hours before testing.

Cells made as described above were found to be permanent, of high sensitivity, of high resistance and positive. The cells were tested frequently over a period of several months. Their resistance was of the order of 5×10^8 ohms in the dark and their light sensitivity was, for all cells, at least 40%, some having as much as 75%.

ATTACHING OF THE ELECTRODES

Small holes, having been made in the insulating forms before the annealing it was an easy task to secure each end of the thread to the form by means of fine wires. These wires then served as electrodes for the cell.

It has been suggested[2] that metallic selenides formed at the expense of the electrodes during the process of annealing played a very important part not only in relation to the conductivity of crystalline selenium but also in bringing about the apparent change of its resistance under the influence of light. Now in the construction of these cells it was just as easy to attach the electrodes after the annealing process had been

[2] Bidwell, Phil. Mag., (5) *20*, p. 178; 1885. Also Chem. News, *52*, p. 191; 1885.

completed as it was to attach them before. Therefore, in some cells the wires were not placed around the selenium until after it was annealed. No difference could be detected between two cells annealed at the same time, one having the electrodes attached and the other not. A number of pairs of cells were built in this manner with the same results. Various metals, Pt, Ni, Cu, etc. were used as electrodes in these pairs of cells, and no difference between cells of the same pair was ever observed.

This type of cell while not adaptable to many of the uses to which selenium cells are put is very useful in certain types of experimental

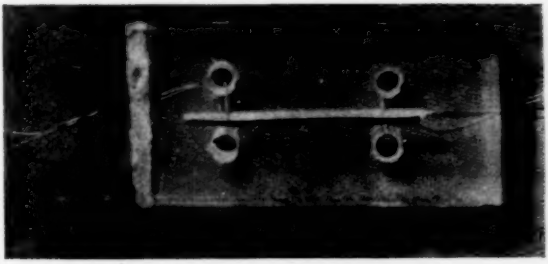


FIG. 1. *Photograph of selenium cell.*

work. The current density at any point is easy to calculate, not only for direct currents but also for alternating currents. The electrodes may be changed easily thus making a study of their characteristic effects possible. The electrodes may be kept in the dark while the central parts of the selenium thread are exposed. This makes it possible to determine whether or not the change of conductivity is due to the action of the light on the body of the selenium or upon the material at or near the electrodes. A number of other uses of this type of cell might be mentioned.

While cells of this type may be made of any size and with various characteristics the particular ones now being used by the author in some experimental work have a length of selenium about 2 cm, a diameter of 0.1 cm and a light sensitivity of 60%. They are mounted upon plate glass and are secured with platinum wires which also serve as electrodes. A photograph of a typical cell is shown in Fig. 1.

PHYSICAL LABORATORY,
LEHIGH UNIVERSITY,
BETHLEHEM, PA.
NOVEMBER, 1927.

Nasal Neurology, Headaches and Eye Disorders. By Greenfield Sluder, M.D., F.A.C.S., Clinical Professor and Director of the Department of Oto-Laryngology, Washington Univ. School of Medicine, 428 pages with 167 illustration, including 2 color plates. Published by C. V. Mosley Co., St. Louis, 1927.

The nasal passages with their numerous accessory sinuses constitute an anatomical structure of intriguing complexity. The air spaces are lined by mucous membrane and surrounded almost everywhere by bone. Through tunnels and channels in this bony wall run the nerves supplying the mucous membrane with pain and tactile sensations, with secretory and other control mechanisms. In addition, there are nerves of neighboring organs whose tunnels or channels through the bones of the head often lie not far from the mucous membrane of the nose or of its accessory sinuses. It is conceivable that in inflammations of the mucous membrane, such as common colds, inflammatory reactions might penetrate to some depth beneath the surface, producing a swelling of the tissues about the nerves, with resultant pressure on the nerves and painful sensations to the patient. On the basis of this essentially reasonable hypothesis, for which, however, there is no definitely substantiating pathological evidence, Dr. Sluder builds the superstructure of what he calls "nasal neurology." He believes that these inflammatory reactions about the nerves in the bones surrounding the upper air passages are frequent occurrences. For reasons best known to himself he has chosen three precise points where the damage is supposed to be done (a. the naso-ciliary nerve in its course near the roof of the nose; b. the spheno-palatine ganglion which lies behind the nasopharynx, and c. the optic and oculomotor nerves in their passage close to the sphenoidal sinus). Finally Dr. Sluder has described in great detail the character and distribution of the pains and aches that are supposed to result from varying degrees of these hypothetical inflammations. The book contains 81 pages of case histories in which the author purports to have found headaches, pains in the face, teeth, tongue, neck, and shoulders which resulted from "nasal neuroses;" and even herpes of the tongue, spasm of the oesophagus, stiffness in the shoulder, asthma, glaucoma, vertigo, nausea, and vomiting are said to have been benefitted by injections into the sphenopalatine ganglion. For a critical review of Sluder's theories the reader is referred to a recent article by H. Burger, Acta Otolaryngologica, *11*, fasc. 2, pp. 221–243.

The book is a tribute to the publisher's art, the numerous anatomical drawings and diagrams being most admirably reproduced, but what can be said of a "Nasal Neurology" that does not mention the sense of smell?

JONAS S. FRIEDENWALD

THE "OLD PANORAMA" PHOTOKYMOGRAPH

By Walter Miles

Scientific work which involves photographic recording whether of deflected beams of light or of a moving shadow in a light field, makes demands for dependable mechanisms to move the film or paper and at the same time provide adequate protection against stray light and incidental exposure. Although a number of elaborate photokymographs have been described[1] and some are now available from the shops of certain instrument makers, it may not be out of place to call attention to what is the most compact and at the same time adaptable device of this sort that the author has ever discovered. This piece of equipment was manufactured as a panoramic camera[2] to handle film five inches wide. The complete equipment includes a large, heavy tripod with a specially devised revolving head and a series of gears by means of which the spring motor built in the camera causes the camera to revolve when the picture is made. The particularly useful part of this commercial equipment is the camera box which is leather covered and of very rugged construction measuring $12 \times 7 \times 4$ inches. The box of course contains the supports for the spool of film, the ground glass focusing screen and the spring-motor with its revolving drum for pulling the film past the exposure slit and for rolling up the exposed portion. The bellows and lens may be easily removed together with the rack and pinion and track carried on the front door. The door itself, however, is very useful in making it possible to close up the camera box completely, so that it will be light-tight for transportation from dark room to wherever it is to be used.

The accompanying illustration, Fig. 1, shows two of these cameras. *A*, represents the camera as secured from the ordinary second-hand dealer, while *B* shows certain simple modifications. The back of the camera is shown open, thus exposing the compartment where the roll of film is held. There is a weak, velvet-covered, leaf spring at *F* for

[1] For a discussion of such apparatus see Garten: Tigerstedt's Handbuch der physiologischen Methodik, *I*, p. 103; 1911.

[2] Manufactured by Folmer and Schwing Division of Eastman Kodak Co. It is their No. 5 Cirkut Camera. The camera is not made at present but I am informed by the manufacturers that they put out about 3,000 of them. As it is a very rugged instrument most of the output must still be in existence in camera exchanges and in the hands of users. The ordinary price charged by these exchanges at present for the complete equipment is around $50.

exerting suitable tension on the film roll during use. At *G* there is a slit 7 mm wide, which during the taking of a panoramic view is at the position here indicated. Focusing is provided for by the ground glass *H*. This is carried in the frame of which the slit *G* is a part; the frame and all can be moved laterally. It is illustrated in *A* in the extreme right-hand position where it would be placed when focusing had been completed. The drum for pulling the film past the slit and rolling up the exposed length is indicated at *I* and a piece of film is shown on the drum.

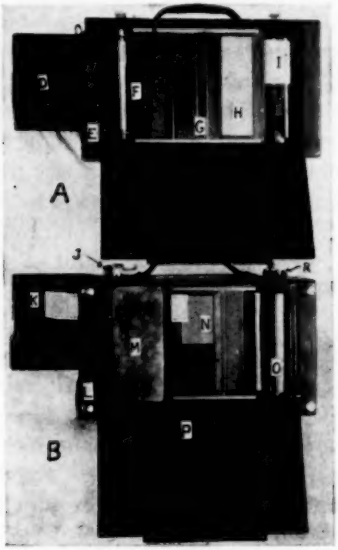


FIG. 1. *A panoramic camera which has been found suitable for use as a photokymograph. A, the camera as supplied for panoramic use. B, the camera modified to use motion picture film.*

The spring motor is at the lower end of the drum. There is a train of gears which extends across the bottom of the camera. These gears are mounted in a pair of metal plates, one of which serves as the bottom of the box. A centrifugal governor is at the position *F* and, in the illustration behind the film compartment. The drum will take film up to 12 cm wide, and it has a circumference of nearly 25 cm (it was made for regular 5 inch spools). The speed of the motor provides for moving film at rates from 1 meter in 2 minutes to 1 meter in 15 seconds. The speed of the drum is satisfactorily regulated by adjusting a small lever

in reference to a graduated scale. This control is at the left of *E* and also the motor is started and stopped by a second lever on the same end of the camera. The spring is wound by a key, the opening for which is directly below the drum. One winding is enough to motivate 3 meters of film at even speed. The relatively large diameter of the drum obviates rapid increase in the rate of film movement due to the winding up of the film on itself.

The drum is clamped to the driving mechanism or made free from it by the knurled head shown on the outside of the box and at the top end of the drum. When the drum is free, the film may be rolled back on the spool by means of the small crank shown above the spool and on the outside of the box. This reroll arrangement is extremely useful in investigations where it is desired to take successive records side by side on the same strip of film.

While the No. 5 Cirkut Camera can be used just as illustrated in *A* of our figure, certain simple modifications naturally suggest themselves. The author has made the changes which are shown in illustration *B*. The wooden block in the space for storing the 9 gears and winding key was removed and a compartment provided in that locality for a spool of fifty feet of regular motion picture film. This roll is held in place by a small shaft *J*. (The roll is pulled out into view in the illustration). The door (*D* and *K*) of this compartment was easily made light-tight and of course is kept closed except when loading film in the compartment. An added door *M* is hinged with leather at the edge near *L* and by a small coiled spring, is held covering the compartment *F* tightly. The large door *P* which forms the back of the camera can thus be opened in ordinary light without exposing any film except the small portion which extends from under the velvet lined edge of door *M*. The focusing screen *N* is the half of *H*, and is moved up to the position illustrated when focusing and then slid down out of the way, so that the light will fall directly on the film. A small spring not shown in the picture holds the glass in whatever position is desired by the operator. The end of the film may then be marked suitably to designate the experiment after which it is drawn to the right and clamped under the small lever hinged at *O* on the drum. The wastage of film by this exposure to light in the room is about 14 cm. When records are being made the film is held snugly against the borders of the windows by a velvet strip which is carried on the door *P*, which door would of course be tightly closed. The recording light may be admitted to the film through the slit *G*, or through the upper half of the frame for the focusing glass.

There is a revolution counter *R*, on the drum, which can be watched or listened to. The camera box can be supported easily on the end of some hood-arrangement and held in place by rubber bands stretched around the four projecting buttons at the corners in *B*. However, a feature of special convenience is offered by the fact that the box is its own support. It can be placed as illustrated in the figure, or stood on either end, (an area 4×7 inches) or placed on its back, directed toward the ceiling. The motor mechanism has proven dependable no matter in what axis the camera is held.[3]

To summarize, the features of merit possessed by this commercial camera used as a photokymograph are as follows: it (1) is a self contained unit, (2) moves film or paper at an even rate, (3) can be used in any axis, (4) is easily transportable, (5) has a wide range of speeds, (6) will take film up to 12 cm in width, (7) can be easily focused without wasting much film, and (8) as a piece of equipment is quite inexpensive.

PSYCHOLOGICAL LABORATORIES,
STANFORD UNIVERSITY,
CALIFORNIA. OCT. 27, 1927.

[3] Dr. Eugene Shen used one of these cameras at Stanford in photographing the eye-movements in reading Chinese. See Miles and Shen, Photographic recording of eye movements in the reading of Chinese in vertical and horizontal axes: method and preliminary results, Journ. Exper. Psychol., *8*, p. 344; 1925, and more recently Dr. M. A. Tinker has used one modified like illustration *B* of Fig. 1, for photographing the eye movements when reading formulas.

AN IMPROVED COORDINATOR[1]

By R. A. Castleman, Jr.

ABSTRACT

A coordinator assembled from a T-square, a drawing board, two slide rules and certain machined accessories has been developed as an improvement on the simple but useful device previously described. The merits claimed for the present design are (a) compactness, simplicity in operation, accuracy and (b) ease of assembly from apparatus readily and cheaply available.

This paper includes a description of the device and some remarks on its design and use.

INTRODUCTION

A plotting device using T-square, drawing board, detachable scales, and suitable indexing and projecting means was described some 23 years ago by R. N. Kotzé.[2] More recently an essentially similar plotting instrument was described by A. H. Sellman.[3]

Both of these instruments, while quite useful for certain purposes, were designed solely for plotting and were not adapted for arbitrary projection to either side of the T-square blade. Only uniform scales were apparently contemplated by Kotzé, and only uniform scales were shown by Sellman, although the latter mentioned the possibility of using logarithmic, reciprocal, trigonometric and uniform scales, singly or in combination.

A year previous to Sellman's publication, the present author described[4,5] the theory and design of a log and semi-log coordinator which had been in use for several years and lacked many of the limitations of the plotting instruments mentioned above. Uniform coordination was included as a by-product.

This simple tool proved so useful that the attempt was made to develop and improvement, based on the same general lines.

DESCRIPTION

A plan view of the device in its final form is shown in the photograph, to which the letters in the following description refer.

[1] Publication Approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

[2] British patent No. 3801 of 1904.

[3] J.O.S.A. & R.S.I., *8*, p. 693; May 1924.

[4] J.O.S.A. & R.S.I., *7*, p. 377; May 1923.

[5] Phys. Rev., (2), *21*, p. 473; April 1923.

It is assembled from a T-square, a drawing board, two slide-rules, and certain machined accessories. To the blade of the T-square is fastened the slide rule *a* whose blade *b* can be adjusted and reversed longitudinally, and fixed, by means of the clamp *c*, in any desired position. The other slide rule *d* is fastened by means of clamp brackets *e* along one side of the drawing board. These brackets *e* are designed to take, without alteration, a slide rule of standard manufacture, so that this can be readily removed for other computing work. The blade *f* and clamp *g* of this slide rule function in a manner similar to *b* and *c*, respectively, above.

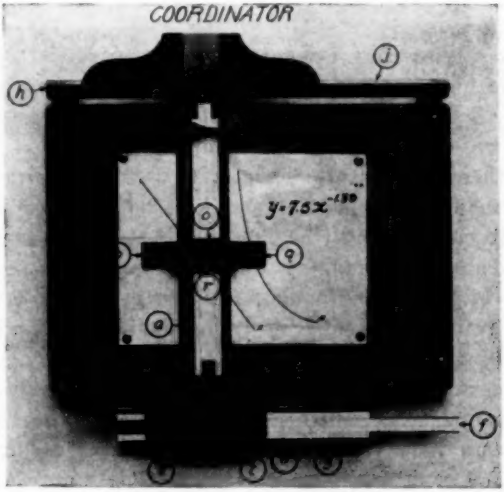


FIG. 1. *Assembled instrument.*

The shoe of the T-square bears on the straight edge of a rail *h* which latter is so fastened to the drawing board that this edge runs (approximately) parallel to the scales on the slide rule *d*. The lower face of this shoe bears lightly on a guide *j*. The shoe is held against the straight edge by a spring which presses against the opposite edge of the rail *h* and which is attached to the lower face of the T-square by means of a post terminating in the nut *k*. The stiffness of this spring is such that, while the operator can freely move the T-square shoe along the rail *h*, there is little danger of the shoe being otherwise disturbed; so that, once having made this setting, other adjustments can be made without further attention to it.

Projecting from opposite sides of the T-square blade are two transparent plates *s* and *t*, the lower face of each being marked with a straight line parallel to an edge of the T-square blade. These lines serve as indices to the scales on the slide rule *d*.

A runner *o* is fitted to slide over the T-square blade and the slide rule *a*. It is held against the face of the slide rule *a* by shoulders that fit in the slide rule grooves; and is guided along the T-square blade by a straight edge held against one edge of this blade by a spring pressing against the other edge. The stiffness of this spring is such as to enable it to function, with regard to the runner *o* and the T-square blade, in a manner similar to the other spring with regard to the T-square and the rail *h*, as described above. This runner *o* has transparent side pieces *p* and *q*, and a transparent central piece *r*.

The side pieces *p* and *q* carry, on their lower faces, a pair of lines that run parallel to an edge of the T-square blade, and whose distance apart equals that between the lines on the plates *s* and *t*. The side plates *p* and *q* also carry two pairs of lines which are prolongations of each other, and which run perpendicular to the edge of the T-square blade. At the intersections of the former pair of lines with either corresponding pair of the latter (the upper pair in the photograph) are drilled small holes for use in plotting, the plain intersections being used in computing.

Across the lower face of the central piece *r*, perpendicular to the edge of the T-square blade, runs a pair of straight lines, whose distance apart equals that between the pair of parallel lines on either plate *p* or plate *q*. These lines act as indices to the scales on the slide rule *a*.

It should be noted that, as the distance between the lines running parallel to the T-square blade on plates *p* and *q* is the same as the distance between those on plates *s* and *t*, either side of the T-square blade may be used indiscriminately. The only additional requirement for arbitrary change from plotting to computing is that the distance between the lines on the plate *r* be equal to that between the parallel pair on either plate *p* or plate *q*.

The blade of each rule carries two sizes of plain log scales, one uniform scale, one reverse log scale, a log sin scale, and a log tan scale; while the body carries three sizes of plain log scales. These furnish practically all scales needed for the three kinds of coordinating here considered. It is to be noted that, since the uniform scales can be "stretched" or "compressed" by any desired amount, the factor used being applied to the slope as read, only one uniform scale is needed.

Two designs have been found useful. One, involving 20 inch slide rules, is particularly serviceable in general laboratory work, while another, which uses 10 inch rules, can be carried around more readily. (The photograph shows the 10 inch design.)

MANIPULATION

The use of the device for the various forms of coordinating should be evident enough, and specific directions seem unnecessary; further than the warning that, in making adjustments, the T-square should be grasped by its shoe, the runner o by its center.

An example of the use of the instrument in one instance may, however, help: In the photograph are shown two curves, both representing the equation $y=7.5x^{-1.30}$. The straight line A represents this equation plotted on logarithmic bases, the log scales D on the two slide rules having been used, and the line erected with the help of the uniform scales L on their blades. Then, by means of the computing feature of the device, corresponding values of x and y were read from the line A, using the same log scales D. Finally, returning to the plotting feature, the curved line B was plotted from these values, using the uniform scales L on the blades, which were conveniently placed and then clamped, as shown.

REMARKS

Several points may be noted here, though some of them can probably be fully appreciated only in actual use.

(1) The apparatus is adapted for use as a log, semi-log and uniform coordinator.

(2) The possibility of arbitrary projection to or from either side of the T-square blade makes possible: (a) avoiding obstructions, such as thumb-tacks; (b) a longer effective bearing to the shoe of the T-square; in the device here described it is always possible to use all this bearing length. Both (a) and (b) are very important when accurate coordination with a reasonably small instrument is desired.

(3) There is a choice of several sizes of log scales, which are related to each other in simple ratio. This makes possible (a) "stretching" or "compressing" the whole plot, thus fitting the data to a convenient size or range; (b) using different size scales for X and Y, thus fitting the data to a convenient shape. Of course, the values of the slope as read from the uniform scales must be corrected by the factor representing the size ratio of the scales used.

(4) It is possible to place a blank sheet in practically any convenient place on the board, and then, choosing the proper scales, adjusting the slide rule blades, and fixing them by means of the clamps c and g, to make the plot fall in any desired place on the sheet.

(5) The fact that the blades of both slide rules, which carry certain log scales, are longitudinally adjustable and fixable, makes possible the treatment of a wider range of data with larger unit scales than would otherwise be feasible. For example, data running from 4.0 to 40.0 could be treated with a *single* log unit on this device, which would not be convenient with scales fixed to the T-square blade or to the board.

(6) The movable uniform scales make it much easier to read slopes and constants and to erect lines of given slope, etc., i.e., they are useful both in interpreting results and in computing.

(7) The coordinator is particularly adapted to the *complete* treatment of equations of the form $y=x\cdot u^n$, $y=b\cdot 10^{mx}$, $y=b_1\cdot e^{m_1 x}$ $y=c\cdot \sin^n x$, $y=c\cdot \tan^n x$, $y=c\cdot \cot^n x$, etc. [Since the relation between the log scales and the uniform scales on these slide rules is that of the Briggian system, the immediate treatment of equations of form $y=b_1 e^{m_1 x}$ may not be so obvious. The easiest method seems to be to use lines of slope $m_1=M\cdot m$, where $M=.4343\ (\log_{10} e)$].

(8) The springs working on the rail h and on the T-square blade, make it easy to use the coordinator in one direction at a time. This makes coordination both easier and more accurate; and is especially important when non-uniform scales, such as log, or (even worse) logs of trigonometric functions, are used.

Obviously, theory and design in back of this simple apparatus are, for many applications, little more complicated than those underlying the slide rules which form its principal constituents. Like these slide rules, its chief claim to attention lies in the uses to which it can be put.

However, as should appear with actual use, even the computing feature of this device can be put to more uses than can most types of slide rule. The addition of the plotting feature, and of the simple and easy access from plotting to computing, make it a very useful tool.

To cite specific examples of its use would probably give an unmerited impression of limited application, besides unnecessarily enlarging this paper. Such examples will doubtless occur to many engaged in laboratory and other similar work, and it seems not unreasonable to expect quite an increase in the number and scope of such potential applications with actual use of the device, as the author has found in his own case.

At the same time, it does not seem necessary to dwell too much on its limitations. That, since "a chain is no stronger than its weakest link," we should expect no more than slide rule accuracy, is obvious. In the present case so much also depends on the accurate rectangularity of the axes at all times that this feature should be carefully looked after, both as to the perpendicularity of the bearing edges of blade and shoe of the T-square and as to the straightness of the bearing edge of the rail *h*.

The author could, of course, not help being primarily influenced in the design by such needs as his own and those of his immediate associates, whose chief problems lie in pure and applied physics. The device should, however, with perhaps slight modifications—principally in the scales—prove useful in such work as chemical research, economic and statistical work, engineering of many other sorts, publication work, etc.

One suggestion deserves particular mention. It is the addition of a reciprocal scale along either *X* or *Y*. Such a scale would undoubtedly be very useful in problems relating to thermionic emission, vapor pressure, chemical reaction rate, etc., and it is to be regretted that slide rules of present construction do not carry them. The following points may, however, be noted: (a) A reciprocal scale is a particularly mean looking affair, very uneven, and probably hard to construct and to use accurately. (b) Several sizes of such scales would be needed, as they connot be "stretched" or "compressed" in a simple manner. (c) Reciprocals can be easily read on this apparatus; the uniform scales may then be used, as pointed out above.

It would be impossible to make detailed acknowledgment here of the many useful criticisms and suggestions from friends and colleagues, but it gives particular pleasure to mention the keen interest shown by, and especially valuable assistance received from, Director Carlile P. Winslow and the staff of the Forest Products Laboratory, Dr. N. E. Dorsey, and Mr. A. D. Adams.

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A NEW SPECTROSCOPIC DEMONSTRATOR FOR THE EXHIBITION OF EMISSION, CONTINUOUS AND ABSORPTION SPECTRA

By H. T. Stetson and H. W. Geromanos

Anyone who has had the teaching of the fundamental principles of spectral analysis in connection with courses in physics, chemistry or astronomy, knows the problem of demonstrating satisfactorily the production of the three distinctive types of spectra. The continuous and the bright line spectra may be shown without much difficulty in the laboratory with the usual stock equipment, but the production of the dark line spectrum is far from so simple an operation.

The present apparatus, as shown in the accompanying illustration, is the result of long experimenting in an endeavor to devise a demonstrator which may be placed in the hands of the students and be relied upon to exhibit instantly and persistently the three principal types of spectra without any assistance from the instructor.

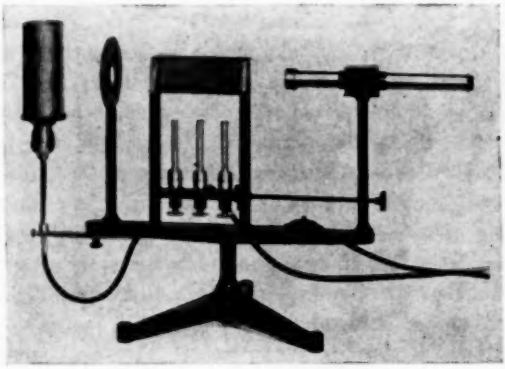

Fig. 1. *Spectroscopic Demonstrator.*

The essential features of the apparatus include a direct vision spectroscope, permanently mounted and directed towards an incandescent lamp of high intensity, mounted on the same support about a foot distant. The lamp is surrounded by a polished nickeled chimney, containing a narrow rectangular window through which the light escapes. A double convex lens focuses an image of the high intensity filament onto the

slit of the spectroscope, giving a brilliantly illuminated continuous spectrum at a turn of the switch.

Between the high powered lamp and the slit of the spectroscope a triplex Bunsen burner is placed. Immediately above the triplex burner, which is of an improved design, is a firmly supported chamber of chrome nickel gauze. This heat resisting bridge carries rods impregnated with a fused super-sodium silicate. The construction of the bridge, together with the special sodiumized rods, is the result of much experiment in a search for a permanent and dependable source of sodium flame easily produced and maintained in gas burners without forced draft. The impregnated rods are held in such a position relative to the burners as to cause the maximum discharge of luminous sodium vapor consistent with long life and dependability. With the electric lamp extinguished, the illumination from the sodium source is sufficient to render clearly visible the two lines λ5390 and λ5396 so characteristic of the emission spectrum of sodium.

If the electric lamp is then turned on by means of the conveniently located switch, the bright emission lines are instantly reversed. The reversed lines show conspicuously dark against the light background of the continuous spectrum and persist so long as both the lamp and sodium source are left in operation.

A particularly valuable feature in the design of the apparatus is the ability to tilt the burners in and out of line with the sodium container by the turning of a knurled knob between fixed stops. The knob is located conveniently near the eye end of the spectroscope and serves to enable the observer to introduce the dark lines into the continuous spectrum or dissipate them at will. The highly satisfactory and unfailing way in which the apparatus has already performed under all sorts of conditions, speaks for its dependability as a piece of demonstration apparatus which can be placed in the hands of a beginning student, with complete assurance on the part of the instructor as to its performance and dependability.

For the excellence of its appearance and workmanship, credit is due to the manufacturers, The Eastern Science Supply Co., Boston, Massachusetts.

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